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METEOROLOGICAL CONDITIONS ACCOMPANYING MIRAGES IN THE SALT LAKE DESERT.

BY

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ABSTRACT.

Meteorological conditions commonly or invariably accompanying mirages in the Salt Lake Desert are here described, and the descriptions illustrated with photographs and instrumental records, where such are available and pertinent. Extended observation indicates that mirages are a standard and predictable phenomenon in the Salt Lake Desert. Less extended field studies show that similar meteorological conditions accompany mirages in all other American deserts.

INTRODUCTION.

Annoying, serious, or tragic difficulties with mirages and similar optical phenomena in the Salt Lake Desert have been recorded by many field workers since October 1, 1776, when the expedition led by Silvestre Veléz de Escalante noted a "lake" in the distance, hastened their steps, and "found that what we had thought to be water was in some places salt, in others saltpeter, and in still others tequesquite (caliche)" (1).[2]

General appearance and behavior of mirages in the Salt Lake Desert were clearly understood by scouts, stage-drivers, and (some) military men at least as early as 1866.[3] Problems of desert observation were noted by G. K. Gilbert during field studies prior to 1885; partial solutions of these problems as they affect geological observations (2) and aeronautical operations (3) have been published recently. Humphreys' general summary of atmospheric optics (4), originally based on data available prior to 1920, but subsequently augmented, contains specific references to the Salt Lake Desert.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

[3] The writer is indebted to the late William H. Jackson for lucid descriptions of mirages seen by him in 1866 near Fish Springs (Fig. 2) on the old stage road from Salt Lake City to Sacramento.

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Field observations here reported were made during the years 1942 to 1946, inclusive, while the writer was stationed at Dugway, Utah, and assigned to problems of chemical meteorology.

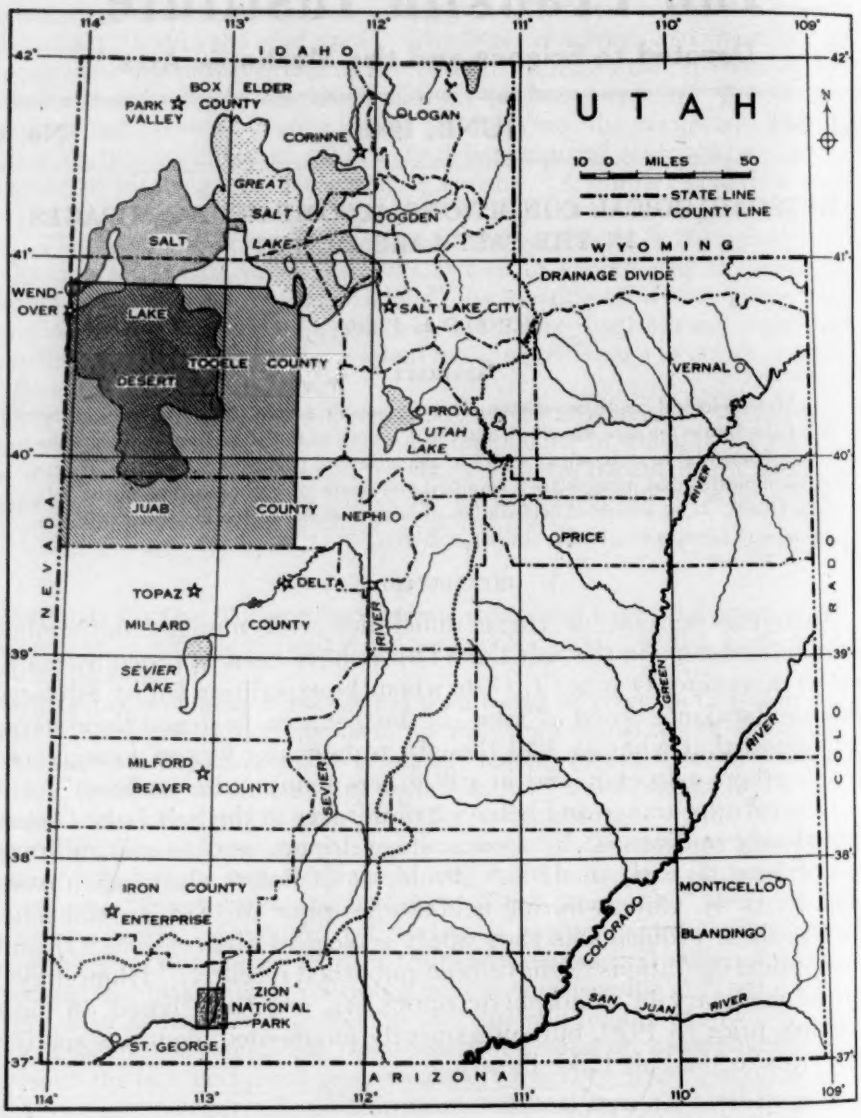

FIG. 1. Index map, showing general location of the Salt Lake Desert with respect to major hydrographic and political divisions of Utah. Intensive field studies were conducted in the southern part of this desert (section lined); numerous field reconnaissances, augmented by reports from cooperative weather observers and airways stations (indicated by stars), indicate that conditions noted in the main area also prevail in the named western counties of Utah.

GEOGRAPHIC SUMMARY.

The Salt Lake Desert is an area of ancient lake beds and fault-block mountains located west and south of Great Salt Lake, in Utah (Fig. 1). About half of the area is occupied by salt deposits, which

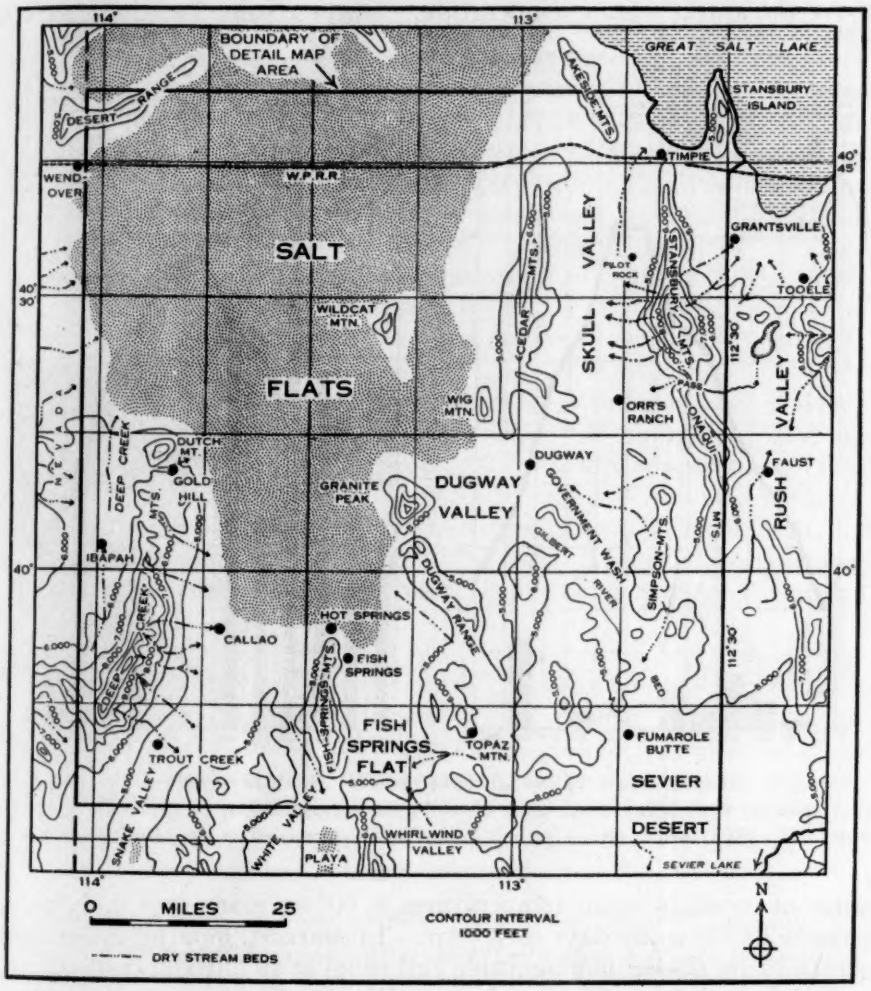

FIG. 2. Topographic map of area of intensive study. Elevation of Great Salt Lake is approximately 4200 ft.; the Salt Flats are about 25 ft. higher.

comprise one of the largest level surfaces in the world, and also one of the most barren; a third of the area is floored with lake-bed clays, in places covered by recent wash from bordering highlands; the remainder consists of desert mountains, many of them "inselbergs." Place names

and topographic features pertinent to this study are shown in Fig. 2 (5).[4]

CLIMATE.

The Salt Lake Desert is a region of climatic extremes. At many flatland stations, the highest summer temperature is more than 120° F. above the lowest winter temperature. In these same locations, annual

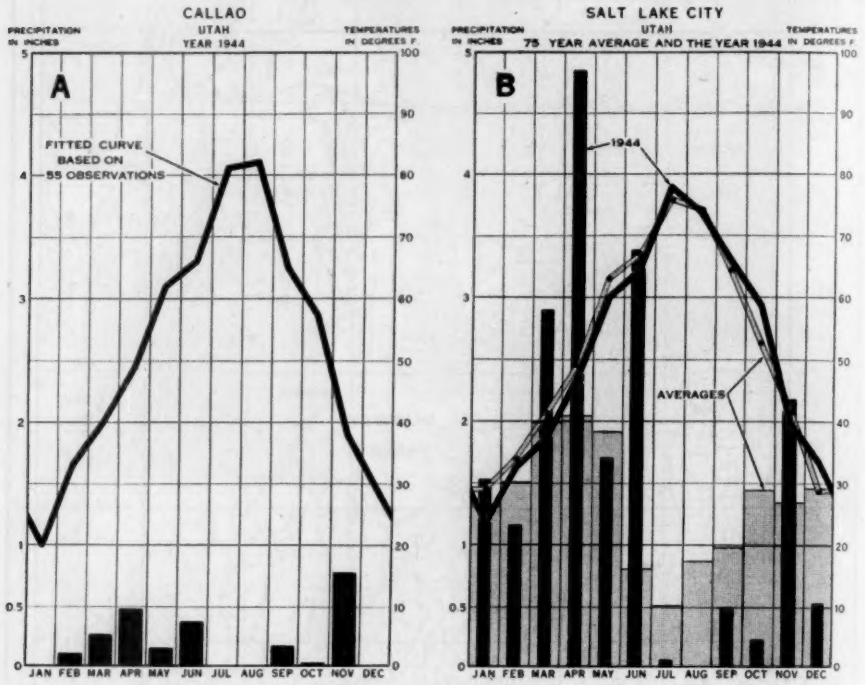

FIG. 3. Climatic charts for the Salt Lake Desert. The 1944 record for Callao is shown at A, monthly mean temperature being shown by the curve, monthly total rainfall by the bar scale. The 1944 climatic record for Salt Lake City is superposed on the 75-yr. average in B.

range of monthly mean temperatures is 60° or more, and daily range exceeds 20° on many days each year. In contrast, monthly mean temperatures are similar in magnitude and trend at all flatland stations, and annual variations are slight. A typical desert climatic chart constitutes Fig. 3 (*A*); comparison of annual and long-term data is shown in Fig. 3 (*B*).

Rainfall in this region is scanty, and so erratic in its areal and sea-

[4] Map modified from the U. S. C. and G. S. *Salt Lake City* sectional aeronautical map, much civil, and all military, culture being omitted. Local inquiry as to present roads and military restrictions is advised before entering this area. Geographic and geologic features of the region are competently described in ref. 5.

sonal distribution that 1-yr. records give little indication of the actual amount or distribution. Long-term records show that rainfall has a major spring maximum, a minor fall maximum, and a summer minimum. At most desert stations, the rainfall recorded on some days exceeds the total precipitation for some years. A 1-yr. record of rainfall at a desert station is shown in Fig. 3(*A*); comparison of rainfall for the same year with the 75-yr. average, at Salt Lake City, is shown in Fig. 3(*B*).

Dryness of this region at all seasons is caused by the barrier effect of surrounding mountains. An appreciable part of the incoming air is from the north Pacific. The lower strata of this incoming air, in crossing the Cascades and Sierras, are forced upward and chilled, so that some of the contained water vapor is condensed, and falls as rain or snow on the west flanks of the ranges. In winter, inflow of air is inhibited by atmospheric subsidence over the Great Basin ("The Great Basin High"), surface manifestation of which is a lateral outflow of cold air. Only strong disturbances can enter the region, at low levels, in winter.

In summer, the same area is occupied by a thermal low, in which heated air ascends, carrying with it evaporated moisture. This air mixes with the upper winds (above 15,000 ft.) before condensation takes place, and is carried eastward. In consequence, much moisture evaporated in the Salt Lake Desert is removed from the area, and eventually increases the humidity of areas far to the east. Potential evaporation, at many places in the Salt Lake Desert, is five or more times the actual precipitation.

Average aeronautical visibility (visual range) in the Salt Lake Desert is not only good, but is considerably better than might be concluded from study of the observational data, which, until recently, placed all visibilities of over ten miles in a single class. Dense fogs of as much as ten days' duration are not uncommon in winter, during atmospheric subsidences related to "Great Basin High" regimes, but their statistical effect is offset, in large part, by "runs" of twenty to forty days in summer, when, over the desert, "there isn't a cloud in the sky all day." [5]

VISIBILITY.

Desert visibility, like most other desert phenomena, is characterized by extremes. Visibility changes in clear or only slightly cloudy desert weather follow a definite, but not always simple, pattern. Maximum visibility commonly occurs at, or very shortly after, sunrise, at which time stratigraphic and structural details in mountains 90 miles distant may be plainly visible to the unaided eye. Visibility then declines slowly, reaching a minimum at about 3 p.m. (sun time), when visibilities of from ten to twenty miles are commonly reported. Visibility improves

[5] See ref. 3. Summaries of atmospheric conditions in this area, by months and years, as far back as 1872, are contained in "Climatological Data—Utah Section" published monthly, with annual summaries, by the U. S. Weather Bureau.

slowly thereafter, but at sunset has not regained its morning value. Sunset visibilities of from forty to sixty miles are commonly reported. When no marked change takes place in air-mass characteristics (*as measured locally*), successive morning visibility maxima normally show a definite, but small, decline.[6]

Within the limitations of current theories and available instruments, the day-to-day decline in visibility within a single air-mass qualitatively and quantitatively parallels the rate of increase of *luftplanktons* in the lower strata of this air mass. This increase in *luftplanktons* is in part due to convectionally raised dust, a natural desert phenomenon, and in part caused by local smelting operations, notably at Salt Lake City, Provo (Fig. 1) and Tooele (Fig. 2) (6).

Field studies, however, show no simple relation between the diurnal cycle of visibility changes and the *luftplankton* content of the lower atmosphere, particularly at very low relative humidities.[7] Definite qualitative and quantitative relations were noted between diurnal changes in the reciprocal of visibility and:

(*a*) Surface temperature,
(*b*) Reciprocal of relative humidity,
(*c*) Lapse rate in the surface stratum,
(*d*) Fluctuations in this lapse rate,
(*e*) Magnitude of convective activity, and
(*f*) Magnitude of shimmer, or "optical haze."

All of these variables are interdependent, within a single air mass, and all are manifestations, usually with a time lag (of about 2 hr.), of diurnal changes in incident solar radiation. Provided maximum relative humidity is low, effect of changes in the temperature of the air as a whole, within the range of temperatures normally encountered in meteorological work, on the optical properties of the air, is slight. Under desert (and most other) conditions, when specific humidity remains constant, relative humidity declines as air temperature increases, and *vice-versa*.[8] When maximum relative humidity is low, effect of relative humidity, and of changes therein, on visibility, is slight.

Field measurements of lapse rates in the Dugway area show that

[6] This "decay of visibility" on successive days, in the same air mass, is competently detailed in ref. 3, p. 185. Similar occurrences have been noted by the writer in the Ajo area, Arizona, and in the Alpine Basin, Texas.

[7] Relative humidities below 10 per cent are not uncommon in the desert at the time of maximum surface temperature (usually 2 p.m. sun time). Although values of 3 and 4 per cent have been recorded at Dugway in a number of instances, accuracy of these values is not guaranteed, because of shortcomings of psychrometric instruments and tables at high altitudes and very low humidities.

[8] Specific humidity is defined as the mass of water vapor in a given mass of moist air, and is usually expressed as grams of water per kilogram of moist air.

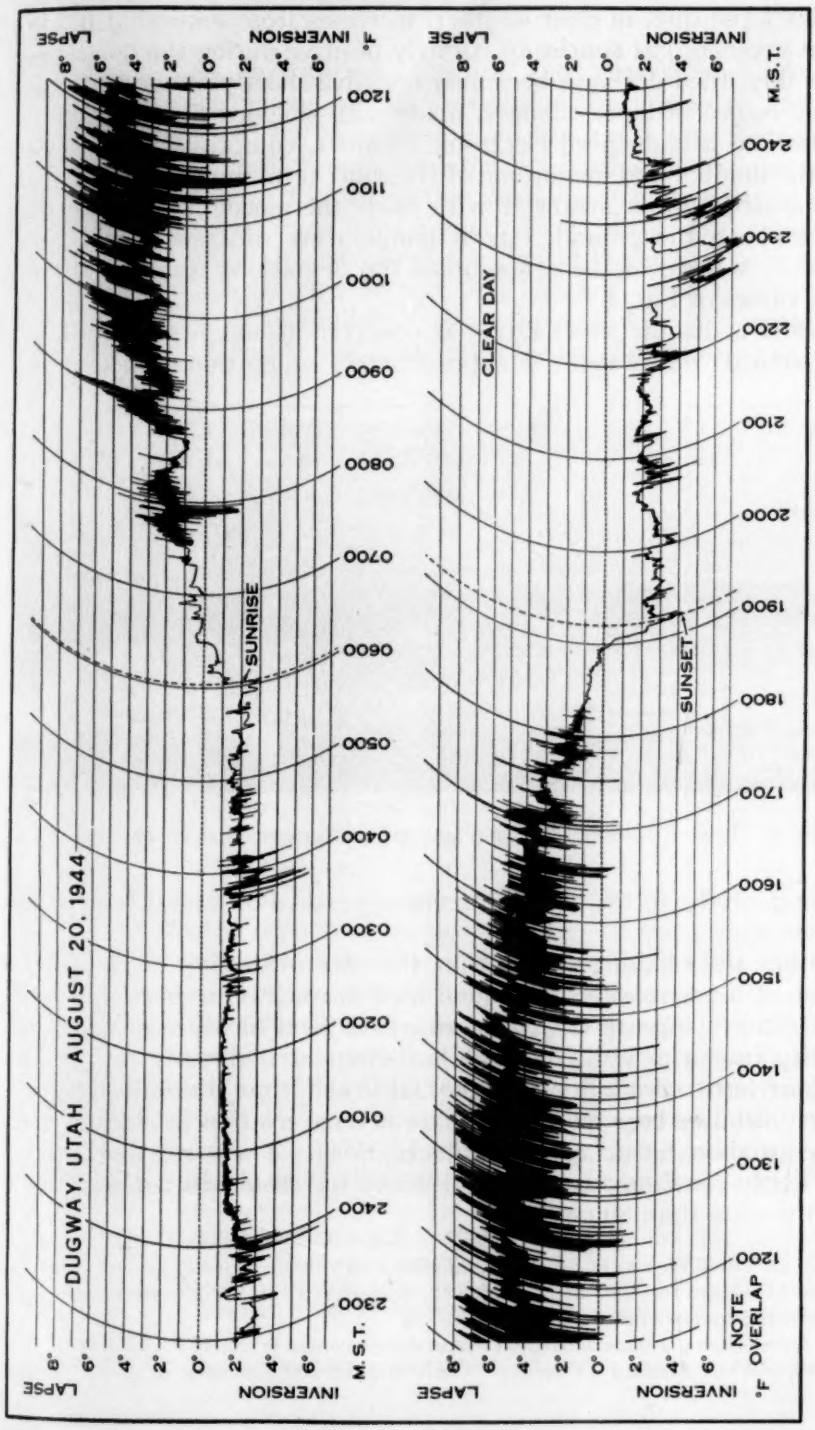

FIG. 4. One-day record of changes in lapse rate at Dugway, Utah. Lower thermocouple was 25 cm. above the ground; upper couple, 2 m. above the surface.

average lapse rate, in clear weather, increases from somewhat negative (slight inversion) at sunrise to strongly positive during the hottest part of the day, then declines, becoming negative shortly before sunset. A sample record of lapse changes, made with shielded fine-wire thermocouples and a high-speed electronic recorder, comprises Fig. 4. Note here the number and magnitude of the short-term fluctuations.[9] If the same instrument is arranged with both thermocouples at the same height above the ground, lateral temperature differences will be recorded. A trace of these resembles the deviations from the 10-min. mean values of Fig. 4.[10]

When a distant small target is observed telescopically, its lateral and vertical "dance rate" is approximately proportional, in frequency

FIG. 5. Typical "oasis and palm tree" mirage, photographed near Wendover, Utah.

and magnitude, to the changes in either vertical or lateral temperature difference.

Other tests having shown that the rate of decline in the concentration of a gas released at ground level is roughly proportional to the lapse rate, it appears that lapse rate is a good measure of convective activity (not a new discovery), that shimmer is directly proportional to short-term variability in lapse rate, and that diurnal changes in desert visibility bear an approximate inverse relation to shimmer, and hence to short-term lapse rate fluctuations. These relationships are sufficiently consistent to permit visibility forecasts with an accuracy of slightly more than 80 per cent.

[9] Lapse rate is the rate of temperature decline with altitude above the surface, commonly expressed in either degrees Fahr. per 1000 ft., or degrees Cent. per kilometer. A negative lapse rate is commonly called an inversion.

[10] Development of these differential temperature recorders is largely the work of Dr. S. W. Grinnell, now of Stanford University. Constructional and operating data are contained in ref. 7.

INFERIOR MIRAGE.

Introduction.

Travelers' tales, exploration accounts, and some serious scientific reports from desert regions contain descriptions of interesting and elusive oases, surrounded by palm trees, even in areas where the palm is not indigenous, and sometimes populated by dancing houris and other atypical desert fauna. The subjective portions of mirage reports, although usually real to those who recount them, are beyond the scope of this paper.[11] Objective nature of the typical "oasis and palm tree" mirage report is indicated by the fact that it can be photographed. A typical example, near Wendover, Utah, comprises Fig. 5.[12] Although inferior mirages of many varieties have been reported, two general

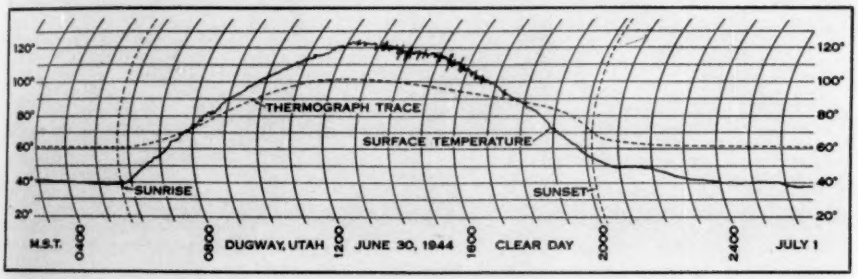

FIG. 6. Twenty-four hour record of temperatures at ground surface and at 5 ft. above the surface, during typical mirage weather.

classifications, based on the thermal state of the lower atmosphere, include the objective portions of all known instances.

Lapse Mirage.

The lapse mirage occurs when the air close to the ground is intensely heated relative to that a few inches or feet above the effective surface. This classification includes the familiar "water on the highway" mirage as well as the common "oasis and palm trees" type (Fig. 5).

Field measurements indicate that the minimum lapse permitting an inferior mirage, in large flat areas in clear sunshiny weather, is about 5° F. in the first foot above a completely barren surface (salt flats). Where vegetation is present, this lapse must occur in the foot above the effective surface, which, in the case of grasslands, is approximately two-

[11] Clinical thirst is normally accompanied by physical desiccation, disturbed body chemistry, and considerable fever. These are commonly augmented by the physiological effects of exhaustion and starvation. In consequence, accounts of things seen during arduous desert journeys are somewhat distorted by the effects of disordered perceptions and by misinterpretations of perceived stimuli.

[12] Additional photographs of mirages are contained in ref. 8 (see opposite p. 144) and ref. 9 (see opposite p. 480).

thirds the height of the grass above the ground. Similar corrections are necessary for areas of slight topographic relief. Because of the "chimney effect" of great relief, mirages are normally absent from badlands and other areas of considerable relief, in which extensive heated air strata cannot normally develop.

Characteristic temperature measurements, during "mirage weather" in the Salt Lake Desert, are shown in Fig. 6. Here, surface tempera-

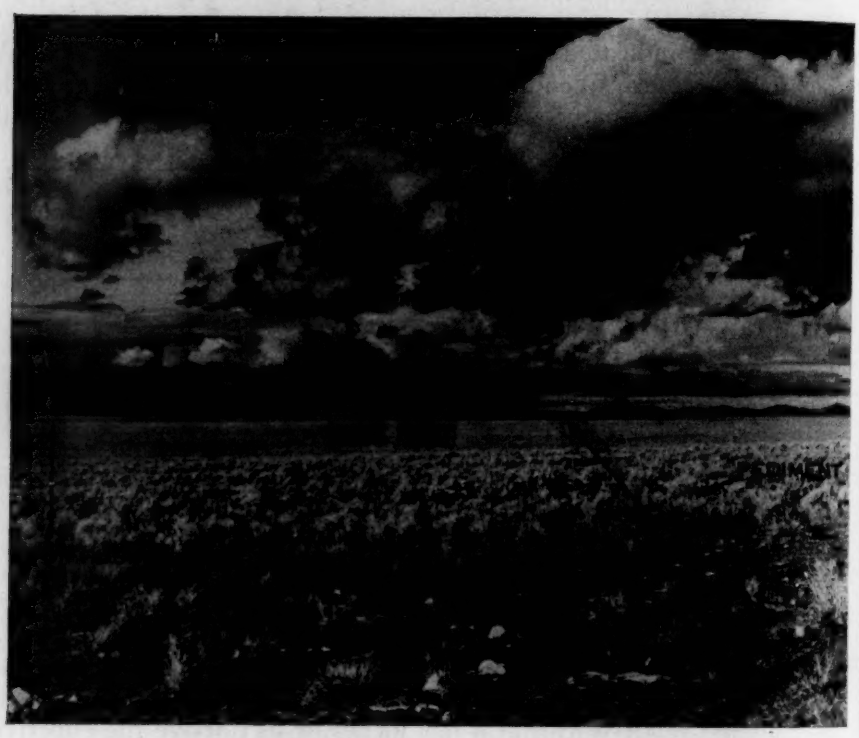

FIG. 7. Onset of mirage conditions in Skull Valley on December 23, 1945, 30 min. after end of storm. Mountains at extreme right are on Stansbury Island; pinnacle at right is Pilot Rock (Fig. 2), about 8 miles from the camera; mountains at left are part of the Lakeside Range, about 15 miles from the camera. Note small mirage between camera and Pilot Rock; and thick diffusing stratum (shimmer layer) between camera and Lakeside Mountains. Dark line on horizon in center is the surface of Great Salt Lake, possibly accompanied by mirages.

tures, measured with a fine-wire thermocouple and recorded electronically (solid line), are compared to air temperatures recorded on a standard thermograph, 5 ft. above the surface, in a Weather Bureau shelter (dashed line). Notable here is the morning change from inversion to lapse, the correspondence of the afternoon "jitters" in the surface temperature curve with the period of maximum shimmer, and the late afternoon change from lapse to inversion.

Weak mirages appeared in salt flat areas northwest of Dugway (Fig.

2) about 35 min. after sunrise on this date; and strong mirages appeared quite suddenly over clay flats (sparse vegetation) at about 8:30 a.m. Mirages remained over all level areas until about 6:15 p.m., when those over the clay flats dissolved quite rapidly. Some of the mirages over barren salt flats persisted until a few minutes after sunset.

During the hottest portion of the day, tongues of mirage extended upward from the flats onto the lower slopes of some of the pediments fringing the mountains. Where field measurements were possible, it was found that these tongues of mirage coincided with updrafts of extremely hot dry air ("anabats"); and that, in general, they persisted only when the updraft was both thin and much warmer than the air directly above it.[13]

Conditions of lighting and atmosphere are normally unfavorable for effective photography of the onset of mirage conditions. In the winter season, however, during the clearing immediately following a storm, such photography is possible. One such example comprises Fig. 7. Here, 30 min. after the cloud cover "broke" over the end of the valley, a fairly thick diffusing stratum (left) had already developed, and a weak mirage was present on the clay flats in the center of the valley. Within an hour, the lower slopes of the Lakeside Mountains, and all of Stansbury Island had "greyed out"; and the mirage was considerably larger, producing an appearance of water from the horizon (actual water) to the base of the pediment, from which the photograph was taken.

Observed lapse mirages over water require about the same lapse condition as those over land, but this need not be produced by insolation, and frequently is not. Because of the high specific heat of water, relative to air, requisite lapse conditions are seldom attained during these parts of the year when average mean temperatures are increasing (late winter, spring, and early summer); but such conditions are present at most times while average mean temperatures are declining (late summer, fall, and early winter). Because extreme lapse may exist over water even at night, "moonlight mirages" are theoretically possible. Field observations disclosed many nights of distinctly abnormal visibility, but no clear-cut instances of nocturnal mirages.

Forecasting of over-water mirages on the basis of lapse rates appears justifiable, but confirmation of these forecasts was found difficult in many instances, as it was not found possible to distinguish between a distant weak mirage over water, and a distant haze layer viewed through shimmer. In general, mirages may be expected over water in most instances when the air a few inches above the water surface is five or more degrees (F.) warmer than that 5 ft. above the water; and are almost invariably absent when the air 5 ft. above the water is at nearly the

[13] Although based on measurements of less than 10 per cent of observed "mirage tongues" in any given day, this appears to be a standard condition, and has been observed also on inclined rock surfaces in the Red Desert of Wyoming, and on the "Flatirons" near Boulder, Colorado.

same temperature as the water. When lapse over water is between the above limits, over-water visibility is usually poor, but definite identification of mirages is not possible.[14]

As might be expected, any disturbance which locally decreases the lapse rate also locally disturbs any mirage which may be present. Chief natural disturbances are cloud shadows and dust devils (**10**). Artificial disturbances include extensive vehicular traffic, explosions, irrigation, and artificial refrigeration. When the disturbance ceases, the mirage usually "heals" in a matter of a few minutes; and when the cause of lapse rate decline is migratory (as a cloud shadow), the "hole" created by it in the mirage is also migratory.

Inversion, Subsidence, or Ponding Mirages.

Similar in appearance and behavior to the lapse mirage, but present only in extremely cold weather, and rarely even then, is the inversion mirage, which appears as a sheet of water in a topographic "low."

Atmospheric conditions accompanying such a mirage are shown diagrammatically in Fig. 8. Although there appears to be no theoret-

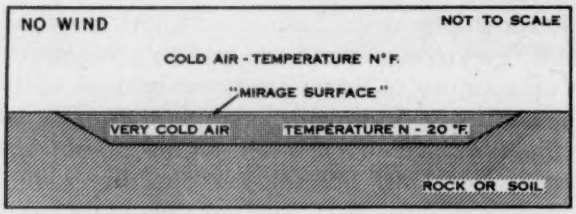

FIG. 8. Atmospheric conditions accompanying inversion mirages.

ical reason why inversion (also known as subsidence or ponding) mirages should not be present whenever very cold air drains into a depression ("ponds"), this type of mirage has only been observed when general air temperatures were far below zero, and the ponded air was twenty or more degrees colder.

These mirages have been observed only at "false dawn," when there was no wind. The boundary between the cold air (above) and the very cold air (below) is quite sharp, one field measurement showing a difference of 15° F. in 1 in. (vertically). Inversion mirages dissipate immediately at sunrise, as does the accompanying extreme inversion, and are destroyed by relatively minor mechanical disturbances, such as a wind of less than 3 mph., the turbulence produced by driving a jeep through them, or the explosion of a half stick of dynamite. Unlike lapse mirages, inversion mirages do not reform rapidly if disturbed.

[14] These statements are based on conditions at Great Salt Lake. Almost identical conditions prevail over the northern part of the Gulf of California, where mirages over the land are most common in summer, and are replaced by mirages over the water in winter.

Field investigations disclose that inversion mirages are formed only in blind basins in the paths of strong air drainages ("katabats") during windless subzero weather, and are not always found even under these rather restrictive conditions.

Rare Types.

At various heights over desert terrain, usually within 2000 ft. of the surface, sheets of "water" are occasionally noted from descending aircraft, or are observed from adjacent mountains. These have the same general appearance as the familiar inferior mirage which commonly is accompanied by steep lapse. In one extreme example, airport operations were disrupted because the entire field appeared submerged, except for the top of the control tower, when observed from a plane coming in for a landing. All airport features were clearly visible from a greater height, as well as from a considerably lesser height.

A few observations indicate that a change in lapse rate occurs at approximately the level of the "water," and suggest that this is simply an elevated inferior mirage.[15] Field observations from mountainsides disclose that these "elevated mirages" are visible through only a very narrow vertical range (such as 15 ft.), and that they are usually visible for only a short time (such as 30 min.) at any given elevation. Slow vertical migration is strongly suspected, but was not conclusively demonstrated by field evidence.

SUPERIOR MIRAGE.

Superior mirages, usually consisting of one or more inverted images of a distant object (which may be below the horizon); in many instances alternating, in "paper doll" fashion, with erect images; are not frequently reported from the Salt Lake Desert. Although superior mirages occur less frequently than inferior mirages, a large part of their apparent rarity is due to non-observation; the sites from which they are most commonly visible being visited rarely.

Relatively simple superior mirages, usually consisting of single inverted images of section houses and similar structures along the railroad at the north end of Skull Valley (*Map*, Fig. 2: *View*, Fig. 7), are occasionally seen, most frequently in early morning, from points ten or fifteen miles upvalley (south). Images are usually distorted, poorly defined, positionally unsteady, and chromatically altered (red in the object appears as brown in the image; white as bluish-green). Life of a superior mirage in this area is seldom as much as an hour. Frequency of occurrence is approximately twice a month; most common time of occurrence is between false dawn and one hour after sunrise. In no

[15] These lapse rate changes take place in too short a vertical distance to be reported with any hope of correctness by any standard radiosonde. Mountain observations, likewise, are subject to some or much error because of mountainside air circulations.

observed or reported instance has a superior mirage coexisted with an inferior mirage in this area, although both have occurred on the same day.

More complicated superior mirages are seen relatively frequently from the north base of Granite Peak (Fig. 2), usually to the north and northwest. Most common mirage of this type is an image of the train going across the desert upside-down. In most instances, the image is multiple, with vertical "paper doll" duplication. Positional unsteadiness, with *jump* being about four times *weave*, is common, as is vertical expansion. Such superior mirages are most common between false dawn and one hour after sunrise, and may be accompanied, just before disappearance, by a nearby inferior mirage.

Moving bands of lights, sometimes in as many as six vertical rows, usually unevenly spaced, are occasionally seen at night in this location. These are probably the same phenomenon, although it was not found possible, even by use of a theodolite, to determine whether the images were erect or inverted.

Rarely (perhaps three times a year), a complicated mirage topography is seen northeast of Wildcat Mountain (Fig. 2) by observers at Granite Peak. This, locally called "Guthman's Towers," is substantially identical to the mirage topography reported by Scoresby from the Davis Strait area.[16] This is invariably an early morning phenomenon, usually vanishing at sunrise. The real topography responsible for this mirage is a series of low hills near Knolls station, south of the railroad (Fig. 2) and near the east edge of the Salt Flats.

Vertical duplication of truck headlights eight or ten miles from the observer was noted in early evening at many points in the Salt Lake Desert, and caused considerable embarrassment to various Military Police charged with traffic counts. About half an hour after five sets of headlights were noted on the road, one truck would pass the M.P. station.

Meteorological investigations disclosed that a superior mirage was invariably accompanied by a definite change in the lapse rate at no great height above the surface. Simple superior mirages were commonly accompanied by a temperature inversion less than 500 ft. above the surface: complex superior mirages were usually accompanied by a complex thermal stratification of the air below the 500-ft. level. Appearance and disappearance of the superior mirage coincided approximately in time with that of the inversion. In general, the mirage and the thermal stratification of the lower atmosphere were of about equal complexity, although limitations of the instruments used preclude any layer-for-layer and image-for-image correlation.

It is notable that inversions, in this area, set in just before sunset, in clear weather, at low levels, and increase in elevation and complexity until shortly after sunrise, when they are dissipated by the upward

[16] See ref. 4, p. 473, Fig. 166. See also included bibliography.

growth of a lapse layer, produced by solar heating of the ground surface and of the air adjacent to it.

FATA MORGANA.

Although Fata Morgana mirages, such as are fairly common in the Straits of Messina, Toyama Bay, Hecate Strait, and the Gulf of Peñas,[17] are very infrequently reported from the environs of Great Salt Lake, many apparently occur there unseen.

Several mirages of this general type were observed from the north-

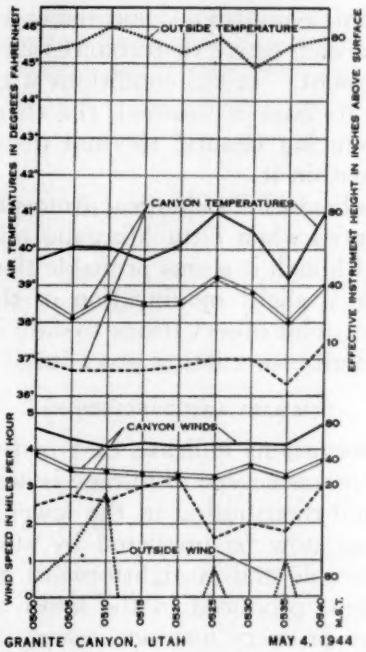

FIG. 9. Wind and temperature conditions within a canyon air drainage.

west shore of Stansbury Island (Fig. 2), and consisted of weirdly-distorted images of mountain terrain, probably adjacent ranges (Stansbury and Lakeside Mountains), although certain identification was impossible.

In contrast to mirages of other types, Fata Morgana images change from minute to minute, and not infrequently have peripheral color fringes, much like those surrounding Brocken Spectres. Complicating the general picture, in many instances, are wisps of "steam fog" rising from the lake water.

[17] Respectively between Italy and Sicily, on the northwest coast of Honshu, on the west coast of British Columbia, and on the Chilean coast between Chiloe Island and the Straits of Magellan.

All observed Fata Morgana displays occurred in early morning, when general air temperatures were not only low, but considerably lower than the temperature of the lake water, when wind speed was very low or zero, and when air drainages from adjacent mountains were strong, and somewhat colder than the static air above the lake.

These conditions produce, at the lake shore, a "two-layer" situation in the lower atmosphere, similar to that commonly accompanying simple mirages. A short distance offshore, this changes to a "three-layer" condition, as basal air is heated by contact with the relatively warm lake water. When this occurs, the basal air stratum, in contact with the lake water, is in unstable equilibrium, and tends to rise. This density readjustment produces convective disturbances largely confined to the coldest air stratum present. Initial conditions within this stratum are shown in Fig. 9. As its base is warmed, the thermal stratification is locally disrupted, producing chaotic thermal distribution (and hence density distribution) within it.

It is notable that substantially identical atmospheric conditions were present in most instances when Fata Morgana mirages were observed in other locations. Although it seems probable that the Fata Morgana can be predicted from a slight modification of the standard weather forecast, the number of joint observations to date is too small to justify establishment of a definite forecasting procedure.

SUMMARY AND CONCLUSIONS.

The foregoing observations indicate that in the Salt Lake Desert, and probably elsewhere, each type of mirage is accompanied by a definite pattern of thermal distribution in the lower atmosphere. These thermal conditions can now be predicted by standard weather forecasting methods, or simple and straightforward modifications thereof.

Density distributions produced in the lower atmosphere by these thermal conditions are in every instance compatible with those called for by current general theories of mirage genesis.[18]

A small amount of additional research, using instruments now extant, although not as yet in general use, should not only refine current theories of mirage formation, permitting quantitative descriptions, but may well lead to the development of dependable procedures for predicting, to a high degree of accuracy, at least 24 hours in advance, the visibility conditions at any place on the earth's surface.

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(1) HERBERT S. AUERBACH, "Father Escalante's Journal with Related Documents and Maps," *Utah Hist. Quart.*, Vol. II, p. 74 (1943).

[18] Salient parts of these theories are more than a half century old, and are, in general, strengthened by recent increases in observational data. For summary of general theories, see ref. 4, pp. 467–475; for detailed descriptions of the various thermal states of the atmosphere, and of the changes therein, see ref. 10, pp. 50–137.

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Pneumatic Barker Cuts Log Waste (*Compressed Air Magazine*, Vol. 53, No. 3).—A significant contribution to the conservation of forest products has apparently been made by the Weyerhaeuser Timber Company, in the State of Washington, with its new air-operated log barker.

Logs intended for processing into plywood have generally been turned in a modified engine lathe while a cutting tool removed the bark. This treatment does the job satisfactorily, but it takes away with the bark around 5 per cent of the wood. Aside from this direct loss of wood, the bared outer surface of the log is torn to some extent and must be smoothed before continuous plywood sheets can be obtained by the subsequent "peeling" operation with a cutting tool. In recent years, use has been made of a hydraulic method that loosens the bark by means of high-pressure water jets. The Weyerhaeuser development represents an effort to accomplish the same result in a simpler and less expensive way.

The pneumatic equipment removes the bark by the compressive and shearing action of a rotating wheel that is pressed tight against the surface of the log as the latter turns in a giant lathe. Pressure is obtained by mounting the wheel on the end of a piston that works in an air cylinder. The apparatus is controlled by one man, who rides a carriage that moves the length of the log as the bark is progressively removed.

The debarker is designed on the simple principle that less force is required to crush bark than to crush wood. When the applied force is sufficient, the bark separates at its point of contact with the cambium, which lies between the bark and the wood, thus producing wood-free green bark as one product and bark-free logs as the other. This is accomplished with the inexpensive equipment at low operating cost. Air pressure of about 50 psi. is used, and in the Weyerhaeuser plant is drawn from the regular supply lines extending to various parts of the mill. The average barking time for a log 8 ft. long and 40 in. in diameter is $1\frac{3}{4}$ min., which is about the same as with previous facilities.

Because of the clean removal of the bark it is possible to utilize the entire log. In the case of fir logs, which are about 12 per cent bark, the latter was formerly considered of little or no commercial value. Now it is converted into ingredients that enter into the manufacture of plastics, insectides, magnesite flooring, rubber compounds, and many other products. It is processed in the Weyerhaeuser Silvacon plant at Longview, Wash., where a huge plywood plant is likewise located. From the bark is also obtained glue extender, of which large amounts are used in fabricating plywood. Thus the new equipment provides more wood for plywood veneer, and the bark contributes its natural properties to help make the plywood durable and weather-proof.

The pneumatic barker represents another forward step in the program undertaken to eliminate the waste that formerly characterized the timber-working industry. Sawdust, wood refuse, bark, and mill ends, that once accumulated in great piles and for which there was no market, are now being converted into valuable by-products by research engineers.

The new barker is the result of years of experimentation by the Weyerhaeuser research staff under the direction of C. C. Heritage, the firm's technical adviser. Patents have been applied for, and the company intends to make the process available to the lumber industry generally. Indications are that it will be applied mainly for debarking large logs.

R. H. O.

APPLICATIONS OF TENSOR ANALYSIS TO ELASTICITY AND PIEZOELECTRICITY.*

BY

JACOB HARRY JURMAIN, M.S.[1]

The subject of elasticity has been covered very completely in the standard English texts, for example, Love (1),[2] Sokolnikoff (2), Timoshenko (3). All of them at least mention tensor notation in passing, and one or two use a form of it in their mathematical analysis, but none presents the subject in a consistent development in terms of tensor analysis.

The only book which actually uses the tensor calculus as the analytical tool in the development of the mathematical theory of elasticity is "Les Tenseurs en Méchanique et en Élasticité," by Brillouin (4), and this book is not available in English.

Most developments on this subject are rather complex, involving the introduction of a number of related metric tensors. These could undoubtedly better be combined to produce a simpler over-all formulation. The tendency to use a number of interdependent functions in place of one in this type of analysis is a weakness, since it reduces the generality and simplicity of form which are two of the main reasons for using tensors. No doubt much of the prejudice against the use of tensors, which is so prevalent among mathematical physicists, derives from this source.

In the subject of piezoelectricity, the tensor notation has been used to a limited extent by Lawson (5), and Atanasoff and Hart (6).

Except for Brillouin (4), none of the authors mentioned above has used more than the notation alone. Although the Einstein summation convention is used to good purpose by most, no distinction is made between covariance and contravariance. Thus it can be seen that the idea of using tensor analysis in the development of elasticity is not a new one, but it should at the same time be obvious that in order to show its true effectiveness as a tool in this field, something more thorough and consistent is to be desired. This paper, I hope, will be a step in that direction.

A relatively small amount of work has been done in the mathematical analysis of piezoelectricity, and I believe that the use of tensors in this practically untouched field can be very fruitful indeed.

* The paper is an abstract of a research that was conducted at Tufts College in 1947. The author is grateful to Dr. D. E. Spencer for her extremely helpful recommendations.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

In all cases throughout the paper, only small deformations will be considered. The theory of finite deformations is beyond the scope of this investigation, and is not applicable in the piezoelectric effect. By a small deformation is meant a deformation such that Hooke's Law involves no second order terms, that is, the transformation can be considered affine.

The tensor notation and the methods of analysis are taken from Schouten and Struik (7). The rationalized mks. system of units is used throughout this paper.

The most important sources of material used in the development of elasticity are Love (1), and Sokolnikoff (2). The material on piezoelectricity is largely from Cady (8), Heising (9), Lawson (5), and Atanasoff and Hart (6).

In general, this paper presents stress, strain, and their mathematical relationships in a coherent tensor development. It then states the fundamental concepts of piezoelectricity in the same notation. These two major topics are related to Newtonian mechanics in the development of a generalized set of equations of natural motion of piezoelectric media. It is shown that such a set of equations, though presently impossible of complete and general solution, can be made, by discriminating selection of boundary conditions, to offer adequate solutions of practical or theoretical problems of harmonic motion of such media.

I. ELASTICITY.

Stress.

Stress can be shown to be a bivalent contravariant tensor, defined by the equation,

$$dF_i = g_{ij}T^{jk}da_k,$$

where dF_i = force (newton) acting on the element of area da_k,
g_{ij} = metric coefficients,
T^{jk} = stress tensor (newton m^{-2}) caused by dF_i,
da_k = element of area (m^2).

In rectangular cartesian coordinates, stress reduces to a force per unit area, whose components are

$$T^{11} = \frac{dF_1}{da_1}, \quad T^{22} = \frac{dF_2}{da_2}, \quad T^{33} = \frac{dF_3}{da_3},$$

$$T^{12} = \frac{dF_1}{da_2}, \quad T^{21} = \frac{dF_2}{da_1}, \quad T^{31} = \frac{dF_3}{da_1},$$

$$T^{13} = \frac{dF_1}{da_3}, \quad T^{23} = \frac{dF_2}{da_3}, \quad T^{32} = \frac{dF_3}{da_2}.$$

In order to confine the discussion of elasticity to relative displacements, we must place certain limiting conditions upon it. The two

conditions immediately applicable to stress are that the system undergoing the stress be in equilibrium for both moment and force.

Let us first consider the consequences of the condition of force equilibrium. A postulate of the tensor calculus states that if a tensor is transported parallel to itself, the transformation can be expressed in the form:

$$T^{jk}(P') = T^{jk}(P) + \nabla_l T^{jk} dx^l, \tag{1}$$

where the point coordinates are x^l. It is evident from this that the difference between stress components on opposite faces of an elemental volume is

$$- \nabla_l T^{jk} dx^l. \tag{2}$$

The net force creating this stress must by definition be

$$dF_i = - g_{ij} \nabla_l T^{jk} dx^l da_k. \tag{3}$$

Since T^{jk} is defined as acting on the k face, distance between the opposite faces under discussion is dx^k. If the coordinate system is orthogonal, then Eq. 3 can be rewritten as

$$dF_i = - g_{ij} \nabla_k T^{jk} d\tau, \tag{4}$$

where $d\tau$ is an element of volume (m^3). If a body force acts throughout the volume, then if A_i/τ be defined as the force per unit volume (newton m^{-3}),

$$dF_i = A_i - g_{ij} \nabla_k T^{jk} d\tau. \tag{5}$$

In conditions of force equilibrium, the total force is zero, or

$$g_{ij} \nabla_k T^{jk} - \frac{A_i}{\tau} = 0. \tag{6}$$

This reduces, in the case of rectangular cartesian coordinates, to

$$\frac{\partial T^{ik}}{\partial x^k} - \frac{A_i}{\tau} = 0, \tag{6a}$$

which, when expanded, is the set of force equilibrium conditions commonly used.

The condition that the system be in a state of moment equilibrium requires that the stress tensor be symmetrical. This can be shown in the following manner.

Moment is a univalent covariant vector defined by the equation,

$$M_i = \frac{g^{lj}}{\sqrt{g}} \oint_s e_{ijk} dF_l dx^k, \tag{7}$$

where dx^k = an incremental moment arm in an axial direction,
dF_l = an incremental force,

g = determinant of g_{ij}, and

$$e_{ijk} = \begin{cases} 1, \text{ if } ijk \text{ is a positive permutation of 123,} \\ -1, \text{ if } ijk \text{ is a negative permutation of 123,} \\ 0, \text{ if } ijk \text{ has any two indices equal.} \end{cases}$$

In equilibrium,

$$M_i = 0.$$

Substituting for dF_l from Eq. 1,

$$\begin{aligned} 0 &= g^{lj} \oint_S e_{ijk} dx^k g_{lm} T^{mn} da_n \\ &= \overset{j}{\underset{m}{\delta}} \oint_S e_{ijk} dx^k T^{mn} da_n. \end{aligned} \tag{8}$$

Application of Gauss's Theorem,

$$\oint_S v^i da_i = \int_V \frac{1}{\sqrt{g}} \frac{\partial(\sqrt{g} v^i)}{\partial x^i} d\tau$$

(where S is the closed surface bounding the volume V), to Eqs. 7 and 8, yields

$$\begin{aligned} M_i &= \overset{j}{\underset{m}{\delta}} \int_V \frac{1}{\sqrt{g}} \frac{\partial \sqrt{g}(e_{ijk} dx^k T^{mn})}{\partial x^n} d\tau \\ &= \overset{j}{\underset{m}{\delta}} \int_V \frac{1}{\sqrt{g}} \Big[\sqrt{g} e_{ijk} T^{mn} \frac{\partial dx^k}{\partial x^n} + \sqrt{g} e_{ijk} \frac{\partial T^{mn}}{\partial x^n} dx^k \\ &\qquad + e_{ijk} dx^k T^{mn} \frac{\partial \sqrt{g}}{\partial x^n} \Big] d\tau. \end{aligned} \tag{9}$$

Since the volume V is not generally zero,

$$\sqrt{g} \overset{j}{\underset{m}{\delta}} e_{ijk} T^{mn} \frac{\partial dx^k}{\partial x^n} + \sqrt{g} \overset{j}{\underset{m}{\delta}} e_{ijk} dx^k \frac{\partial T^{mn}}{\partial x^n} + \overset{j}{\underset{m}{\delta}} e_{ijk} dx^k T^{mn} \frac{\partial \sqrt{g}}{\partial x^n} = 0. \tag{10}$$

But in force equilibrium conditions, by Eq. 6,

$$\frac{\partial T^{mn}}{\partial x^n} = 0$$

(if higher order terms are neglected and body forces equal zero). Therefore

$$\sqrt{g} \overset{j}{\underset{m}{\delta}} e_{ijk} T^{mn} \frac{\partial dx^k}{\partial x^n} + \overset{j}{\underset{m}{\delta}} e_{ijk} dx^k T^{mn} \frac{\partial \sqrt{g}}{\partial x^n} = 0. \tag{11}$$

In orthogonal coordinate systems

$$\frac{\partial dx^k}{\partial x^n} = \begin{cases} 1, & \text{for } n = k, \\ 0, & \text{for } n \neq k. \end{cases}$$

Thus we may state

$$e_{ijk}T^{jk} = 0,$$

or

$$T^{jk} = T^{kj}. \tag{12}$$

But tensor symmetry is an invariant property under coordinate transformations and therefore Eq. 12 holds in general if there are no unbalanced external forces.

Strain.

If surface and volume forces are applied to a deformable body, its parts undergo relative displacements which increase until equilibrium is reestablished by the internal forces. The changes in relative positions of the parts of a body under stress are called strains.

Under the influence of applied force, a point P is displaced to a point P'. Call the displacement vector $s^i(P)$. In the same deformation, a nearby point, $\overset{(1)}{P}$, is displaced to a point $\overset{(1')}{P}$. Its displacement vector is $s^i(\overset{(1)}{P})$. The relative displacement of the two points is then

$$s^i(\overset{(1)}{P}) - s^i(P).$$

If we now apply the postulate of parallel transport, we observe that

$$s^i(\overset{(1)}{P}) - s^i(P) = \nabla_k s^i(P)dx^k = \Delta s^i. \tag{13}$$

Expanding, we obtain:

$$\Delta s^i = \nabla_1 s^i(P)dx^1 + \nabla_2 s^i(P)dx^2 + \nabla_3 s^i(P)dx^3, \tag{14}$$

or,

$$\Delta s^i = \frac{\partial s^i}{\partial x^1}dx^1 + \frac{\partial s^i}{\partial x^2}dx^2 + \frac{\partial s^i}{\partial x^3}dx^3 + \Gamma^i_{l_1}s^l + \Gamma^i_{l_2}s^l + \Gamma^i_{l_3}s^l. \tag{15}$$

The terms $\dfrac{\partial s^i}{\partial x^j}$ are called deformation components.

Let us define two tensors S_{ij} and R_{ij} in the following way:

$$S_{ij} = \frac{1}{2}\left(g_{ik}\frac{\partial s^k}{\partial x^j} + g_{jk}\frac{\partial s^k}{\partial x^i} \right), \tag{16}$$

and

$$R_{ij} = \frac{1}{2}\left(g_{ik}\frac{\partial s^k}{\partial x^j} - g_{jk}\frac{\partial s^k}{\partial x^i} \right). \tag{17}$$

It is now evident that

$$R_{ij} + S_{ij} = g_{ik}\frac{\partial s^k}{\partial x^j}. \tag{18}$$

Thus the sum of the two tensors is precisely a measure of the deforma-

tion components already defined. If the higher order Christoffel functions are neglected, a justifiable step in a small deformation, the transformation can be described completely in terms of the tensors S_{ij} and R_{ij}.

Further investigation will disclose that the tensor R_{ij} is related to that portion of the deformation which is a local translation or rotation of a segment of the deformed body, but not a relative displacement. If we consider only conditions of equilibrium for such local rotation and translation, the R_{ij} terms vanish.

The S_{ij} terms, which define only that portion of the deformation which is a relative displacement, are called strains. Thus S_{ij} is known as the *strain tensor*.

Although no extension of the idea will be attempted in this paper, it is interesting to note that both stress and strain, in any practical problem, include both symmetric and antisymmetric components. If moment equilibrium of a volume element is not maintained, that is, rotation is permitted, stress can be broken into symmetric and antisymmetric components.

To maintain point-to-point continuity of the transformation when a volume element is strained, a certain amount of rotation and translation must take place in addition to pure relative displacements. In this case, the antisymmetric portion of the strain, R_{ij}, does not vanish.

Thus it is apparent that in any practical consideration, although Hooke's Law is satisfied, there must be another relation linking the antisymmetric portions of stress and strain. This condition results in the equations of compatibility which will be considered in the next section.

A physical interpretation may be placed upon strain in the following way: It can be shown that for $i \neq j$, S_{ij} is a measure of one half the angular deformation produced by an applied stress; and that the diagonal terms of the strain matrix are the relative changes in length.

Stress-Strain Relations.

1. Hooke's Law.

Hooke has shown experimentally that small deformations of an elastic body can be described by an affine relationship between stress and strain. In tensor notation this may be expressed as

$$T^{ij} = c^{ijkl} S_{kl}, \tag{19}$$

with the inverse relation,

$$S_{kl} = b_{klij} T^{ij}. \tag{20}$$

It is evident from these two equations that

$$c^{ijkl} b_{klmn} = \overset{ij}{\underset{mn}{\delta}}, \tag{21}$$

where $\overset{ij}{\underset{mn}{\delta}}$ is the quadruply indexed Kroniker Delta. The elastic constants, c^{ijkl}, exhibit symmetry in indices such that

$$c^{ijkl} = c^{jikl} = c^{ijlk} = c^{jilk} = c^{klij} = c^{lkij} = c^{lkji} = c^{klji}.$$

2. Equations of Compatibility.

From the points of view both of the physical and the mathematical aspects of displacements, it would be desirable that s^i, the displacement vector, be continuous and single valued throughout the deformation. This will put restrictions upon the values of S_{ij}, due to the fact that pure deformation with no translation of the various portions of the body would produce point-to-point discontinuities through the body. The equations stating these restrictions are called the equations of compatibility.

One method of stating the displacement equation is

$$\overset{(1)}{s^j}(P) = s^j(P) + \Delta s^j = s^j(P) + \int_P^{(1)P} ds^j$$

$$= s^j(P) + g^{ji}\int_P^{(1)P} S_{ik}dx^k + g^{ji}\int_P^{(1)P} R_{ik}dx^k. \quad (22)$$

If the last member of this equation be integrated by parts,

$$\int_P^{(1)P} R_{ik}dx^k = R_{ik}(\overset{(1)}{x^k} - x^k) - \int_P^{(1)P} (\overset{(1)}{x^k} - x^k)\frac{\partial R_{ik}}{\partial x^i}dx^i. \quad (23)$$

It is a simple matter to show that

$$\frac{\partial R_{ik}}{\partial x^i} = \frac{\partial S_{il}}{\partial x^k} - \frac{\partial S_{kl}}{\partial x^i}. \quad (24)$$

Substituting back into the displacement equation, we obtain

$$\overset{(1)}{s^j}(P) = s^j(P) + g^{ji}R_{ik}(\overset{(1)}{x^k} - x^k)$$

$$+ g^{ji}\int_P^{(1)P}\left[S_{il} - (\overset{(1)}{x^k} - x^k)\left(\frac{\partial S_{il}}{\partial x^k} - \frac{\partial S_{kl}}{\partial x^i}\right)\right]dx^l. \quad (25)$$

Since we wish the displacement, $\overset{(1)}{s^j}(P)$, to be independent of the path of integration, the argument under the integral must be an exact differential. The necessary and sufficient condition is

$$\frac{\partial}{\partial x^j}\left[S_{il} - (\overset{(1)}{x^k} - x^k)\left(\frac{\partial S_{il}}{\partial x^k} - \frac{\partial S_{kl}}{\partial x^i}\right)\right]$$

$$- \frac{\partial}{\partial x^l}\left[S_{ij} - (\overset{(1)}{x^k} - x^k)\left(\frac{\partial S_{ij}}{\partial x^k} - \frac{\partial S_{kj}}{\partial x^i}\right)\right] = 0. \quad (26)$$

In orthogonal coordinate systems this equation reduces to

$$\frac{\partial^2 S_{ij}}{\partial x^k \partial x^l} - \frac{\partial^2 S_{kj}}{\partial x^j \partial x^l} - \frac{\partial^2 S_{il}}{\partial x^k \partial x^j} + \frac{\partial^2 S_{kl}}{\partial x^i \partial x^j} = 0. \tag{27}$$

Since i, j, k, l can take on only three values, two at least must be equal. The non-trivial conditions $i = j$, $i = l$, $j = k$, or $k = l$, all yield the same equation:

$$\frac{\partial^2 S_{ii}}{\partial x^k \partial x^l} = \frac{\partial^2 S_{ki}}{\partial x^i \partial x^l} + \frac{\partial^2 S_{il}}{\partial x^k \partial x^i} - \frac{\partial^2 S_{kl}}{(\partial x^i)^2}. \tag{28}$$

The six non-identical equations obtainable from the above relation are known as the equations of compatibility.

Elastic Symmetry.

It is evident that, in the most general case, Hooke's Law can be expressed by two sets of nine equations, each equation containing nine terms.

In many cases, however, particularly those involving crystalline substances, the materials under consideration exhibit some degree of inherent structural symmetry. Such symmetry simplifies the stress-strain relations to an extent dependent upon the degree of symmetry. Tensor algebra can be used to obtain the simplified relations with a minimum of difficulty. The method is to determine all independent coordinate transformations with respect to which the elastic constants of the medium are invariant, to compare the relations obtained, and to simplify.

Thus *in a medium exhibiting orthonormal symmetry*, there are two independent transformations; these are any two of the three simple reflections:

$$\begin{array}{lll} x^{1'} = -x^1, & x^{1'} = x^1, & \\ x^{2'} = x^2, & x^{2'} = x^2, & \text{and} \\ x^{3'} = x^3, & x^{3'} = -x^3, & \end{array} \qquad \begin{array}{l} x^{1'} = x^1, \\ x^{2'} = -x^2, \\ x^{3'} = x^3. \end{array}$$

Hooke's Law can be expressed as follows:

$$\begin{aligned} T^{11} &= c^{1111}S_{11} + c^{1122}S_{22} + c^{1133}S_{33}, \\ T^{22} &= c^{2211}S_{11} + c^{2222}S_{22} + c^{2233}S_{33}, \\ T^{33} &= c^{3311}S_{11} + c^{3322}S_{22} + c^{3333}S_{33}, \\ T^{23} &= T^{32} = 2c^{2323}S_{23}, \\ T^{13} &= T^{31} = 2c^{1313}S_{13}, \\ T^{12} &= T^{21} = 2c^{1212}S_{12}. \end{aligned}$$

In a homogeneous isotropic medium, the above reflections, plus three independent rotations, will yield all the relations necessary for the derivation of the equations characterizing an isotropic medium.

Three possible rotations are:

$$x^{1'} = x^1, \quad x^{2'} = x^3, \quad x^{3'} = -x^2, \qquad x^{1'} = x^2, \quad x^{2'} = -x^1, \quad x^{3'} = x^3,$$

$$\text{and} \qquad x^{1'} = x^1\frac{\sqrt{2}}{2} + x^2\frac{\sqrt{2}}{2}, \quad x^{2'} = -x^1\frac{\sqrt{2}}{2} + x^2\frac{\sqrt{2}}{2}, \quad x^{3'} = x^3.$$

In this case Hooke's Law reduces to

$$\begin{aligned}
T^{11} &= c^{1111}S_{11} + c^{1122}(S_{22} + S_{33}),\\
T^{22} &= c^{1111}S_{22} + c^{1122}(S_{11} + S_{33}),\\
T^{33} &= c^{1111}S_{33} + c^{1122}(S_{11} + S_{22}),\\
T^{23} &= T^{32} = 2c^{2323}S_{23},\\
T^{13} &= T^{31} = 2c^{2323}S_{13},\\
T^{12} &= T^{21} = 2c^{2323}S_{12},
\end{aligned}$$

where

$$c^{2323} = \tfrac{1}{2}(c^{1111} - c^{1122}).$$

If the medium is homogeneous and isotropic, let us consider the values of the elastic constants throughout the body to be as follows:

$$\mu = c^{2323} \qquad \text{and} \qquad \lambda = c^{1122}.$$

Now it is easy to obtain the relations

$$T^{ij} = \lambda(S_{11} + S_{22} + S_{33}) + 2\mu S_{ij} \qquad (i = j) \tag{29a}$$

and

$$T^{ij} = 2\mu S_{ij} \qquad (i \neq j). \tag{29b}$$

These are the familiar equations relating stress and strain in a homogeneous isotropic medium.

Similar equations may be obtained for the strain terms.

$$S_{ij} = \frac{T^{ij}}{2\mu} - \frac{\lambda}{2\mu(3\lambda + 2\mu)}(T^{11} + T^{22} + T^{33}) \qquad (i = j), \tag{30a}$$

$$S_{ij} = \frac{T^{ij}}{2\mu} \qquad (i \neq j). \tag{30b}$$

Simple Examples in Elasticity.

We are now in a position to derive the relation between the c^{ijkl} and commonly used moduli of elasticity, and to observe their use in simple cases of elastic deformations.

Example I.—Consider a circular cylinder whose axis is in the x^1 direction, acted upon by a longitudinal stress only. Then $T^{11} = T$, and all other stresses are zero. Thus

$$S_{11} = \frac{T}{2\mu} - \frac{\lambda T}{2\mu(3\lambda + 2\mu)} = \frac{(\lambda + \mu)T}{\mu(3\lambda + 2\mu)},$$

$$S_{22} = S_{33} = \frac{-\lambda T}{2\mu(3\lambda + 2\mu)}.$$

$$S_{ij} = 0 \qquad (i \neq j).$$

Thus

$$\frac{S_{22}}{S_{11}} = \frac{S_{33}}{S_{11}} = \frac{-\lambda}{2(\lambda + \mu)}.$$

Let us define a term σ such that

$$\sigma \equiv \frac{\lambda}{2(\lambda + \mu)}, \tag{31}$$

and a term E, such that

$$E \equiv \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}. \tag{32}$$

Then

$$S_{11} = \frac{T}{E},$$

and

$$S_{22} = S_{33} = \frac{-\sigma T}{E} = -\sigma S_{11}.$$

Thus

$$E = \frac{T}{S_{11}}.$$

E is known as *Young's Modulus* and is the ratio of tensile stress to the extension produced by the stress. Also,

$$\sigma = \left|\frac{S_{22}}{S_{11}}\right| = \left|\frac{S_{33}}{S_{11}}\right|,$$

known as *Poisson's Ratio*, shows the ratio of the contraction perpendicular to the length, to the extension along the length of the cylinder.

The constants λ and μ, already introduced, are known as the *Lamé constants*. They satisfy the relations,

$$\lambda = \frac{E\sigma}{(1 + \sigma)(1 - 2\sigma)} \tag{33}$$

and

$$\mu = \frac{E}{2(1 + \sigma)}. \tag{34}$$

Example II.—Consider next the problem of a parallelepiped acted upon by a pure shear, such that

$$T^{23} = T^{32} = T$$

and

$$T^{11} = T^{22} = T^{33} = T^{12} = T^{21} = T^{13} = T^{31} = 0.$$

Then

$$S_{23} = S_{32} = \frac{T}{2\mu}$$

and

$$\mu = \frac{T}{2S_{32}}.$$

Recalling the fact that S_{32} is a measure of the angular deformation, it is evident, then, that μ is the ratio of shear stress $T^{ij}(i \neq j)$ to the changes in angle, $2S_{ij}(i \neq j)$. μ is called the *modulus of rigidity*, or *shear modulus*.

Example III.—In a body under hydrostatic pressure there is no shear stress, and all compressional stresses are equal. Thus

$$T^{11} = T^{22} = T^{33} = -\rho$$

and

$$T^{ij} = 0 \qquad (i \neq j).$$

Then

$$S_{11} = S_{22} = S_{33} = \frac{-\rho}{3\lambda + 2\mu}$$

and

$$S_{ij} = 0 \qquad (i \neq j).$$

From this,

$$S_{11} + S_{22} + S_{33} = \frac{-\rho}{\lambda + \frac{2}{3}\mu}.$$

Let us define a term K such that

$$K = \lambda + \tfrac{2}{3}\mu. \tag{35a}$$

It can be seen also then that

$$K = \frac{E}{3(1 - 2\sigma)}. \tag{35b}$$

Then

$$K = \frac{-\rho}{S_{11} + S_{22} + S_{33}}$$

and is thus the ratio of compressional stress to the sum of the relative changes in length in the principal directions, or simply, to the change in volume or bulk. K is called the *modulus of compression*, or *bulk modulus*.

It will be observed that K is always positive, thus yielding the condition,

$$\sigma < \tfrac{1}{2}.$$

We may now write the equations for Hooke's Law in a homogeneous isotropic medium in the form:

$$S_{11} = \frac{1}{E}[T^{11} - \sigma(T^{22} + T^{33})],$$

$$S_{22} = \frac{1}{E}[T^{22} - \sigma(T^{33} + T^{11})], \tag{36a}$$

$$S_{33} = \frac{1}{E}[T^{33} - \sigma(T^{11} + T^{22})],$$

and

$$S_{ij} = \frac{1 + \sigma}{E} T^{ij} \qquad (i \neq j). \tag{36b}$$

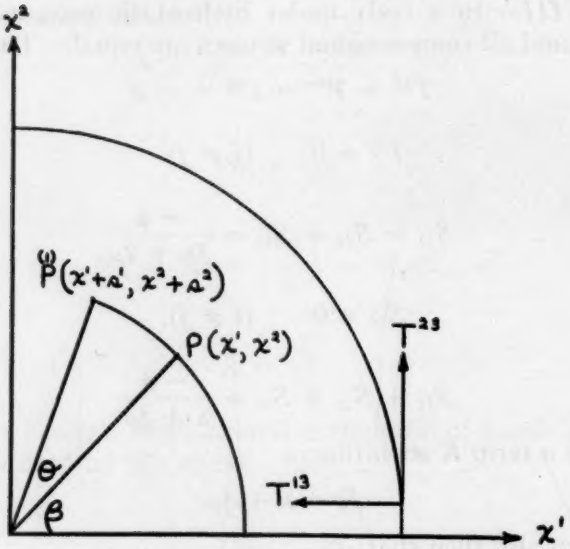

FIG. 1.

Example IV.—Finally, let us consider simple torsion of a circular shaft. We must assume that angular rotation at any cross section will be proportional to the perpendicular distance from the fixed face.

Let the length of the cylinder lie along the x^3 axis. Let

$$\theta = \alpha x^3,$$

where α is the angular rotation per unit length, and θ is the angle through which the cross section to be considered is rotated.

In a cartesian coordinate system, after deformation, a point $P(x^1, x^2)$ will become $\overset{(1)}{P}(x^1 + s^1, x^2 + s^2)$, as in Fig. 1. Then

$$s^1 = r\cos(\beta + \theta) - r\cos\beta = x^1(\cos\theta - 1) - x^2\sin\theta$$

and

$$s^2 = r \sin(\beta + \theta) - r \sin \beta = x^1 \sin \theta + x^2(\cos \theta - 1).$$

If θ is small, $s^1 \sim -x^2\theta$ and $s^2 \sim x^1\theta$. Then

$$s^1 = -\alpha x^2 x^3, \qquad s^2 = \alpha x^1 x^3.$$

From this,

$$\begin{aligned} &S_{11} = 0, \quad S_{22} = 0, \quad S_{33} = 0, \\ &S_{12} = S_{21} = \tfrac{1}{2}(-\alpha x^3 + \alpha x^3) = 0, \\ &S_{13} = S_{31} = \tfrac{1}{2}(-\alpha x^2), \\ &S_{23} = S_{32} = \tfrac{1}{2}(\alpha x^1), \end{aligned}$$

and from Hooke's Law,

$$\begin{aligned} T^{13} &= T^{31} = -\mu\alpha x^2, \\ T^{23} &= T^{32} = \mu\alpha x^1, \end{aligned}$$

and

$$T^{11} = T^{22} = T^{33} = T^{12} = T^{21} = 0.$$

An even simpler method of handling such a problem is provided by transformation to a circular cylinder coordinate system.

$$\begin{aligned} x^{1'} &= r, \\ x^{2'} &= \theta, \\ x^{3'} &= \mathfrak{z}. \end{aligned}$$

Thus $P(x^{1'}, x^{2'})$ after deformation becomes $\overset{(1)}{P}(x^{1'}, x^{2'} + s^{2'})$, where

$$s^{2'} = \alpha x^{3'}, \qquad s^{1'} = 0, \quad s^{3'} = 0.$$

Then

$$S_{1'1'} = S_{2'2'} = S_{3'3'} = S_{1'2'} = S_{2'1'} = S_{1'3'} = S_{3'1'} = 0$$

and

$$S_{2'3'} = S_{3'2'} = \tfrac{1}{2} g_{2'2'}\alpha = \frac{(x^{1'})^2\alpha}{2} = \frac{\alpha r^2}{2}.$$

From Hooke's Law,

$$T^{2'3'} = T^{3'2'} = \mu'\alpha(x^{1'})^2 = \mu'\alpha r^2.$$

The solution obtained by the two methods can easily be correlated. In cartesian coordinates

$$\begin{aligned} |T^{ij}| &= \sqrt{g_{ij}g_{kl}T^{ik}T^{jl}} = \sqrt{g_{11}g_{33}(T^{13})^2 + g_{22}g_{33}(T^{23})^2} \\ &= \mu\alpha\sqrt{(x^1)^2 + (x^2)^2} = \mu\alpha r. \end{aligned}$$

In circular cylinder coordinates

$$|T^{i'j'}| = \sqrt{g_{i'j'}g_{k'l'}T^{i'k'}T^{j'l'}} = \sqrt{g_{2'2'}g_{3'3'}(T^{2'3'})^2} = rT^{2'3'} = \mu'\alpha r^3.$$

But

$$\mu' = c^{2'3'2'3'} = \frac{\partial x^{2'}}{\partial x^i}\frac{\partial x^{3'}}{\partial x^j}\frac{\partial x^{2'}}{\partial x^k}\frac{\partial x^{3'}}{\partial x^l}c^{ijkl}.$$

From the transformation equations

$$r = \sqrt{(x^1)^2 + (x^2)^2},$$

$$\theta = \tan^{-1}\frac{x^2}{x^1},$$

$$\delta = \delta,$$

it can be shown that

$$\mu' = \frac{\mu}{r^2}.$$

Thus

$$|T^{i'j'}| = \mu\alpha r = |T^{ij}|.$$

II. PIEZOELECTRICITY.

Fundamental Equations.

To clarify the terms used in the *piezoelectric equations* below, let us define certain of them which have not already appeared in the paper.

Electric Field Intensity is expressed by E_i (volts m^{-1}),
Electric Flux Density by D^i (coulomb m^{-2}),
Permittivity by ϵ^{ij} (farad m^{-1}),
Polarization by P^i (coulomb m^{-2}), and
Electric Susceptibility by η^{ij} (farad m^{-1}).

They are related in the following ways:

$$D^i = \epsilon^{ij}E_j, \tag{37a}$$

$$P^i = D^i - \underset{(0)}{\epsilon^{ij}}E_j, \tag{37b}$$

where $\underset{(0)}{\epsilon^{ij}}$ is the permittivity of free space, and

$$\underset{(0)}{\epsilon^{11}} = \underset{(0)}{\epsilon^{22}} = \underset{(0)}{\epsilon^{33}} = 8.854 \times 10^{-12} \text{ farad } m^{-1},$$

$$\underset{(0)}{\epsilon^{ij}} = 0 \quad \text{if} \quad (i \neq j),$$

$$\eta^{ij} = \epsilon^{ij} - \underset{(0)}{\epsilon^{ij}},$$

and hence

$$P^i = \eta^{ij}E_j. \tag{37c}$$

Certain materials manifest a property whereby the existence of mechanical stress or strain is directly associated with the existence of an electric polarization within the material, and with an electric field in the surrounding medium.

It has been found (**8**) that this property can be adequately de-

scribed in four equations which we shall call the fundamental piezoelectric equations.

$$P^i = \eta^{ij}E_j + e^{ikl}S_{kl}, \tag{I}$$

is the direct effect of polarization caused by a specified applied field and strain.

A similar equation may be written for the direct effect of polarization resulting from an applied field and stress.

$$P^i = \eta^{ij}E_j + d_{kl}{}^i T^{kl}. \tag{II}$$

The converse effect may also be stated in two equations. With both strain and field prescribed,

$$T^{ij} = c^{ijkl}S_{kl} - e^{mij}E_m. \tag{III}$$

With both stress and field prescribed,

$$S_{ij} = b_{ijkl}T^{kl} + d_{ij}{}^m E_m. \tag{IV}$$

The trivalent tensors e^{ijk} and $d_{jk}{}^i$ are known respectively as the *piezoelectric stress coefficient* and the *piezoelectric strain coefficient.*

The stress T^{ij} in Eq. III above is the total external stress caused by the strain S_{kl} and field E_m. However, an external stress is only indirectly caused by E_m, which, since it acts as a body force, causes an internal stress. The body, due to its rigidity, establishes an equal and opposite external stress which is thus a portion of the measurable stress.

The most important piezoelectric substances are crystalline in nature. The condition which determines in which direction piezoelectric effects appear, is physical asymmetry of axes. That is, any axis along which a piezoelectric effect appears must be such that if the crystal is rotated 180° about an axis normal to the given axis, the rotated crystal cannot be superimposed on the unrotated crystal by a pure translation. Thus the axis considered has a distinct directionality, and piezoelectric constants will be zero except along such axes.

Piezoelectricity in Quartz.

All the applications of piezoelectricity to be considered in this paper will be to quartz crystals. The piezoelectric stress and strain coefficients reduce to the following matrices.

For e^{ijk}:

$$e^{1jk} = \begin{pmatrix} e^{111} & 0 & 0 \\ 0 & -e^{122} & e^{123} \\ 0 & e^{132} & 0 \end{pmatrix}, \quad e^{2jk} = \begin{pmatrix} 0 & e^{212} & e^{213} \\ e^{221} & 0 & 0 \\ e^{231} & 0 & 0 \end{pmatrix}, \quad e^{3jk} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$e^{111} = -e^{122} = -e^{221} = -e^{212},$$
$$e^{123} = e^{132} = -e^{231} = -e^{213}.$$

For $d_{jk}{}^{i}$:

$$d_{jk}{}^{1} = \begin{pmatrix} d_{11}{}^{1} & 0 & 0 \\ 0 & d_{22}{}^{1} & d_{23}{}^{1} \\ 0 & d_{32}{}^{1} & 0 \end{pmatrix}, \quad d_{jk}{}^{2} = \begin{pmatrix} 0 & d_{12}{}^{2} & d_{13}{}^{2} \\ d_{21}{}^{2} & 0 & 0 \\ d_{31}{}^{2} & 0 & 0 \end{pmatrix}, \quad d_{jk}{}^{3} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$d_{11}{}^{1} = -d_{22}{}^{1} = -d_{12}{}^{2} = -d_{21}{}^{2},$$
$$d_{23}{}^{1} = d_{32}{}^{1} = -d_{31}{}^{2} = -d_{13}{}^{2}.$$

And for c^{ijkl}:

$$c^{11kl} = \begin{pmatrix} c^{1111} & 0 & 0 \\ 0 & c^{1122} & c^{1123} \\ 0 & c^{1132} & c^{1133} \end{pmatrix}, \quad c^{12kl} = \begin{pmatrix} 0 & c^{1212} & c^{1213} \\ c^{1221} & 0 & 0 \\ c^{1231} & 0 & 0 \end{pmatrix},$$

$$c^{13kl} = \begin{pmatrix} 0 & c^{1312} & c^{1313} \\ c^{1321} & 0 & 0 \\ c^{1331} & 0 & 0 \end{pmatrix}, \quad c^{22kl} = \begin{pmatrix} c^{2211} & 0 & 0 \\ 0 & c^{2222} & c^{2223} \\ 0 & c^{2232} & c^{2233} \end{pmatrix},$$

$$c^{21kl} = \begin{pmatrix} 0 & c^{2112} & c^{2113} \\ c^{2121} & 0 & 0 \\ c^{2131} & 0 & 0 \end{pmatrix}, \quad c^{23kl} = \begin{pmatrix} c^{2311} & 0 & 0 \\ 0 & c^{2322} & c^{2323} \\ 0 & c^{2332} & 0 \end{pmatrix},$$

$$c^{31kl} = \begin{pmatrix} 0 & c^{3112} & c^{3113} \\ c^{3121} & 0 & 0 \\ c^{3131} & 0 & 0 \end{pmatrix}, \quad c^{32kl} = \begin{pmatrix} c^{3211} & 0 & 0 \\ 0 & c^{3222} & c^{3223} \\ 0 & c^{3232} & 0 \end{pmatrix},$$

$$c^{33kl} = \begin{pmatrix} c^{3311} & 0 & 0 \\ 0 & c^{3322} & 0 \\ 0 & 0 & c^{3333} \end{pmatrix},$$

where

$$c^{1111} = c^{2222},$$
$$c^{2323} = c^{3232} = c^{3223} = c^{1313} = c^{1331} = c^{3113} = c^{3131} = c^{2332},$$
$$c^{1122} = c^{2211},$$
$$c^{1133} = c^{2233} = c^{3311} = c^{3322},$$
$$c^{1123} = c^{1132} = c^{1312} = c^{1321} = c^{3112} = c^{3121} = c^{1213} = c^{1231} = c^{2113}$$
$$= c^{2131} = c^{2311} = c^{3211} = -c^{2223} = -c^{2232} = -c^{2322} = -c^{3222},$$
$$c^{1212} = c^{1221} = c^{2112} = c^{2121} = \tfrac{1}{2}(c^{1111} - c^{1122}).$$

The only non-vanishing permittivity values are ϵ^{11}, ϵ^{22}, and ϵ^{33} where $\epsilon^{11} = \epsilon^{22}$. Thus electric susceptibility has only three values η^{11}, η^{22}, and η^{33}.

The standard axes in quartz are three Euclidian sets, all possessing a common x^3 or Z-axis. The x^3 axis is ordinarily known as the *optical axis* because of its orientation in the direction of optical activity.

The x^1 axes or X-axes are diametral lines connecting opposite corners of the hexagon formed when an idealized crystal is cut in a plane perpendicular to the m faces (**8**, p. 26).

The x^2 axes or Y-axes are respectively perpendicular to the three x^1 axes, and are, consequently, perpendicular to the m faces.

The x^1 axes are also known as *electric axes*, and the x^2 axes are known as *mechanical axes*.

Quartz shows the same properties with respect to each of the three sets of Euclidean axes. The constants given above for quartz hold for all three sets of axes; that is, they are invariant to rotations of $\frac{2\pi}{3}$ radians about the x^3 axis.

In actual practice quartz is customarily used in the form of small plates cut in specific orientations with respect to the fundamental sets of axes. The purpose of these various "cuts," as they are called, are diverse, usually having to do with the characteristics under temperature change of the cuts selected. Thus, cuts such as the AT, BT, and GT produce the least variation in frequency under changes in temperature. Another purpose is to obtain cuts which show a high selectivity of vibrational modes.

Analysis of Motion in Quartz Plates.

In order to set up the equations describing the motion of a piezoelectric body, we shall use the following equations which have appeared previously in this paper:

the stress equation of force equilibrium,

$$F_i = g_{ik}\nabla_j T^{kj}, \tag{6b}$$

where F_i is a body force, or force per unit volume;

the defining equation of the strain tensor,

$$S_{ij} = \tfrac{1}{2} g_{ik}\frac{\partial s^k}{\partial x^j} + g_{jk}\frac{\partial s^k}{\partial x^i}\,; \tag{16}$$

and the third piezoelectric equation,

$$T^{ij} = c^{ij\alpha\beta}S_{\alpha\beta} - e^{\gamma ij}E_\gamma. \tag{III}$$

In addition to these equations we shall use Newton's second law, which can be expressed in the form

$$F_i = g_{ij}\rho\frac{\partial^2 s^j}{\partial t^2}\,, \tag{38}$$

where F_i is again a body force (newton m^{-3}), ρ is the density (kg m^{-3}) of the medium under consideration, and t is the time (sec.).

In orthogonal affine transformations of cartesian coordinates, Eq. 1 becomes

$$F_i = g_{ik}\frac{\partial T^{kj}}{\partial x^j}\cdot \tag{39}$$

Equations 38 and 39 can be combined to obtain

$$g_{ij}\rho \frac{\partial^2 s^j}{\partial t^2} - g_{ik} \frac{\partial T^{kj}}{\partial x^j} = 0. \tag{40}$$

Since we have already limited this portion of the discussion to orthogonal coordinate systems, we may write Eq. 40 in the form

$$\rho \frac{\partial^2 s^i}{\partial t^2} - \frac{\partial T^{ij}}{\partial x^j} = 0. \tag{41}$$

Using Eq. III and substituting for T^{ij} in Eq. 41, we obtain

$$\rho \frac{\partial^2 s^i}{\partial t^2} - c^{ij\alpha\beta} \frac{\partial S_{\alpha\beta}}{\partial x^j} + e^{\gamma ij} \frac{\partial E_\gamma}{\partial x^j} = 0. \tag{42}$$

We may now substitute for $S_{\alpha\beta}$ in Eq. 42 by making use of Eq. 16. This yields the equation

$$\rho \frac{\partial^2 s^i}{\partial t^2} - \frac{c^{ij\alpha\beta}}{2} \left(g_{\alpha\sigma} \frac{\partial^2 s^\sigma}{\partial x^\beta \partial x^j} + g_{\beta\sigma} \frac{\partial^2 s^\sigma}{\partial x^\alpha \partial x^j} \right) + e^{\gamma ij} \frac{\partial E_\gamma}{\partial x^j} = 0. \tag{43}$$

Using the matrices for the elastic and piezoelectric constants in quartz, we can expand Eq. 43 to the following three equations. For $i = 1$,

$$\begin{aligned} \rho \frac{\partial^2 s^1}{\partial t^2} = {} & c^{1111} \frac{\partial^2 s^1}{(\partial x^1)^2} + c^{1212} \frac{\partial^2 s^1}{(\partial x^2)^2} + c^{1313} \frac{\partial^2 s^1}{(\partial x^3)^2} + 2c^{1213} \frac{\partial^2 s^1}{\partial x^2 \partial x^3} \\ & + (c^{1122} + c^{1212}) \frac{\partial^2 s^2}{\partial x^2 \partial x^1} + (c^{1123} + c^{1312}) \frac{\partial^2 s^2}{\partial x^1 \partial x^3} \\ & + (c^{1133} + c^{1313}) \frac{\partial^2 s^3}{\partial x^1 \partial x^3} + (c^{1123} + c^{1213}) \frac{\partial^2 s^3}{\partial x^1 \partial x^2} \\ & - e^{111} \frac{\partial E_1}{\partial x^1} - e^{213} \frac{\partial E_2}{\partial x^3} - e^{212} \frac{\partial E_2}{\partial x^2}. \end{aligned} \tag{44}$$

For $i = 2$,

$$\begin{aligned} \rho \frac{\partial^2 s^2}{\partial t^2} = {} & c^{2112} \frac{\partial^2 s^2}{(\partial x^1)^2} + c^{2222} \frac{\partial^2 s^2}{(\partial x^2)^2} + c^{2323} \frac{\partial^2 s^2}{(\partial x^3)^2} + (c^{2223} + c^{2322}) \frac{\partial^2 s^2}{\partial x^2 \partial x^3} \\ & + (c^{2112} + c^{2211}) \frac{\partial^2 s^1}{\partial x^1 \partial x^2} + (c^{2311} + c^{2113}) \frac{\partial^2 s^1}{\partial x^1 \partial x^3} + c^{2113} \frac{\partial^2 s^3}{(\partial x^1)^2} \\ & + c^{2323} \frac{\partial^2 s^3}{(\partial x^2)^2} + (c^{2233} + c^{2323}) \frac{\partial^2 s^3}{\partial x^2 \partial x^3} - e^{122} \frac{\partial E_1}{\partial x^2} \\ & - e^{123} \frac{\partial E_1}{\partial x^3} - e^{221} \frac{\partial E_2}{\partial x^1}. \end{aligned} \tag{45}$$

For $i = 3$,

$$\rho \frac{\partial^2 s^3}{\partial t^2} = c^{3113} \frac{\partial^2 s^3}{(\partial x^1)^2} + c^{3223} \frac{\partial^2 s^3}{(\partial x^2)^2} + c^{3333} \frac{\partial^2 s^3}{(\partial x^3)^2}$$
$$+ (c^{3211} + c^{3112}) \frac{\partial^2 s^1}{\partial x^1 \partial x^2} + (c^{3311} + c^{3113}) \frac{\partial^2 s^1}{\partial x^1 \partial x^3}$$
$$+ c^{3112} \frac{\partial^2 s^2}{(\partial x^1)^2} + c^{3222} \frac{\partial^2 s^2}{(\partial x^2)^2} + (c^{3322} + c^{3223}) \frac{\partial^2 s^2}{\partial x^2 \partial x^3}$$
$$- e^{132} \frac{\partial E_1}{\partial x^2} - e^{231} \frac{\partial E_2}{\partial x^1}. \quad (46)$$

A set of equations which holds in generalized orthogonal coordinate systems can be developed as follows:

$$g_{ij} \rho \frac{\partial^2 s^j}{\partial t^2} = g_{ij} \nabla_k T^{jk}.$$

This may be rewritten as

$$g_{ii} \left(\rho \frac{\partial^2 s^i}{\partial t^2} - \nabla_j T^{ij} \right) = 0,$$

or

$$\rho \frac{\partial^2 s^i}{\partial t^2} - \nabla_j T^{ij} = 0.$$

Expanding on the covariant derivative, we obtain

$$\rho \frac{\partial^2 s^i}{\partial t^2} - \frac{\partial T^{ij}}{\partial x^j} - \Gamma^i_{jl} T^{lj} - \Gamma^j_{jl} T^{il} = 0.$$

Apply Eqs. 16 and III to obtain

$$\rho \frac{\partial^2 s^i}{\partial t^2} - \frac{c^{ij\alpha\beta}}{2} \left(g_{\alpha\sigma} \frac{\partial^2 s^\sigma}{\partial x^\beta \partial x^j} + g_{\beta\sigma} \frac{\partial^2 s^\sigma}{\partial x^\alpha \partial x^j} \right)$$
$$+ e^{\gamma ij} \frac{\partial E_\gamma}{\partial x^j} - \Gamma^i_{jl} T^{lj} - \Gamma^j_{jl} T^{il} = 0.$$

This last equation is quite general since we have put no restrictions on it as to particular coordinate systems. Thus it includes, among others, the cartesian coordinate system, for which the equations have already been developed.

Applications to Piezoelectric Vibrations.

The generalized equations derived above are far too complex for solution by ordinary methods. It is not the purpose of this paper to enter into any discussion of the complete solution of the system of three differential equations which they present. However, by applying appropriate boundary conditions, we shall see that the standard solutions

for the motion of certain idealized bodies can be obtained from these equations; and we shall attempt to obtain some insight into the methods of applying sophisticated boundary conditions in order to obtain solutions which are very close approximations to actual physical conditions.

In general, those terms involving electric field intensity can be neglected, since at resonance they represent much smaller forces than the elastic stresses.

Example I: Let us first consider the ***longitudinal vibration*** of an infinitely thin rod, which can be expressed by the following boundary conditions:

$$s^1 = f(x^1, t)$$

and

$$s^2 = s^3 = 0.$$

Applying these conditions to Eqs. 44, 45 and 46 above, the only non-trivial expression remaining is

$$\rho \frac{\partial^2 s^1}{\partial t^2} = c^{1111} \frac{\partial^2 s^1}{(\partial x^1)^2},$$

which defines the wave velocity of the motion in the x^1 direction as

$$V_1 = \sqrt{\frac{c^{1111}}{\rho}}.$$

To solve for the frequency of oscillation, we assume that

$$s^1 = g(x^1)h(t).$$

Then

$$\frac{\rho}{h(t)} \frac{\partial^2 h(t)}{\partial t^2} = \frac{c^{1111}}{g(x^1)} \frac{\partial^2 g(x^1)}{(\partial x^1)^2},$$

$$\frac{\partial^2 h(t)}{\partial t^2} = -\frac{m^2}{\rho} h(t),$$

and

$$\frac{\partial^2 g(x^1)}{(\partial x^1)^2} = -\frac{m^2}{c^{1111}} g(x^1).$$

The solutions of these equations are

$$h(t) = A \sin \sqrt{\frac{m^2}{\rho}} t + B \cos \sqrt{\frac{m^2}{\rho}} t$$

and

$$g(x^1) = c \sin \sqrt{\frac{m^2}{c^{1111}}} x^1 + D \cos \sqrt{\frac{m^2}{c^{1111}}} x^1.$$

Thus

$$s^1 = \left(A \sin \frac{m}{\sqrt{\rho}} t + B \cos \frac{m}{\sqrt{\rho}} t \right) \left(C \sin \frac{m}{\sqrt{c^{1111}}} x^1 + D \cos \frac{m}{\sqrt{c^{1111}}} x^1 \right).$$

Now let us apply the boundary condition that the rod be clamped at one end.

$$x^1 = 0,$$
$$s^1 = 0.$$

Then

$$AD \sin \frac{m}{\sqrt{\rho}} t + BD \cos \frac{m}{\sqrt{\rho}} t = 0.$$

Either

$$D = 0,$$

or

$$\tan \frac{m}{\sqrt{\rho}} t = -\frac{B}{A}.$$

The latter case is of less practical interest than the first. Thus

$$s^1 = C \left(A \sin \frac{m}{\sqrt{\rho}} t + B \cos \frac{m}{\sqrt{\rho}} t \right) \sin \frac{m}{\sqrt{c^{1111}}} x^1.$$

Let us assume also that at $t = 0$, $s^1 = 0$. This requires that $B = 0$, and hence that

$$s^1 = AC \sin \frac{m}{\sqrt{\rho}} t \sin \frac{m}{\sqrt{c^{1111}}} x^1.$$

If the other end of the rod is an antinode, then at $x^1 = L$,

$$\frac{\partial s^1}{\partial x^1} = 0.$$

This yields the equation

$$\cos \frac{m}{\sqrt{c^{1111}}} L = 0.$$

But

$$\frac{m}{\sqrt{\rho}} = 2\pi f.$$

Then

$$2\pi f \sqrt{\frac{\rho}{c^{1111}}} L = \frac{(2n+1)\pi}{2}$$

and

$$f = \frac{(2n+1)}{L} \sqrt{\frac{c^{1111}}{\rho}}.$$

Example II: The pure shear problem, with boundary conditions

$$s^1 = f(x^2)g(t)$$

and

$$s^2 = s^3 = 0,$$

yields the differential equation

$$\rho \frac{\partial^2 s^1}{\partial t^2} = c^{1212} \frac{\partial^2 s^1}{(\partial x^2)^2}.$$

If the body is assumed to be clamped at both x^2 faces, and a time boundary is set, that is, $s^1 = 0$ at $x^2 = \pm b/2$ and $s^1 = 0$ at $t = 0$, the solution becomes

$$f = \frac{n}{b} \sqrt{\frac{c^{1212}}{\rho}},$$

where b is the thickness of the plate.

Example III: A more complex problem and one more applicable to actual conditions uses the boundary conditions

$$s^1 = l(x^1, x^2, x^3, t) \tag{47}$$

and

$$s^2 = s^3 = 0.$$

Equations 44, 45, and 46 above reduce to

$$\rho \frac{\partial^2 s^1}{\partial t^2} = c^{1111} \frac{\partial^2 s^1}{(\partial x^1)^2} + c^{1212} \frac{\partial^2 s^1}{(\partial x^2)^2} + c^{1313} \frac{\partial^2 s^1}{(\partial x^3)^2} + 2c^{1213} \frac{\partial^2 s^1}{\partial x^2 \partial x^3}, \tag{48}$$

$$0 = (c^{2112} + c^{2211}) \frac{\partial^2 s^1}{\partial x^1 \partial x^2} + (c^{2311} + c^{2113}) \frac{\partial^2 s^1}{\partial x^1 \partial x^3}, \tag{49}$$

and

$$0 = (c^{3211} + c^{3112}) \frac{\partial^2 s^1}{\partial x^1 \partial x^2} + (c^{3311} + c^{3113}) \frac{\partial^2 s^1}{\partial x^1 \partial x^3}. \tag{50}$$

Equations 49 and 50 can be combined to show that

$$\frac{\partial^2 s^1}{\partial x^1 \partial x^2} = 0 \tag{51}$$

and

$$\frac{\partial^2 s^1}{\partial x^1 \partial x^3} = 0. \tag{52}$$

Integrate Eq. 51 with respect to x^2 to obtain

$$\frac{\partial s^1}{\partial x^1} = m(x^1, x^3, t). \tag{53}$$

Integrate Eq. 52 with respect to x^3 to obtain

$$\frac{\partial s^1}{\partial x^1} = n(x^1, x^2, t). \tag{54}$$

Thus

$$m(x^1, x^3, t) = n(x^1, x^2, t). \tag{55}$$

But Eq. 55 requires that x^2 and x^3 appear only as constants in Eqs. 53 and 54. Therefore

$$\frac{\partial s^1}{\partial x^1} = h(x^1, t). \tag{56}$$

Integrating with respect to x^1, we obtain

$$s^1 = \int h(x^1, t)dx^1 + g(x^2, x^3, t), \tag{57}$$

which may be written

$$s^1 = f(x^1, t) + g(x^2, x^3, t). \tag{58}$$

Assume now that both f and g are sinusoidal with respect to time.

$$f(x^1, t) = F(x^1) \sin \omega t. \tag{59}$$

$$g(x^2, x^3, t) = G(x^2, x^3) \sin \omega t. \tag{60}$$

Applying Eqs. 59 and 60 to Eq. 48, we obtain

$$-\rho\omega^2[F(x^1) + (Gx^2, x^3)] = c^{1111}\frac{\partial^2 F(x^1)}{(\partial x^1)^2} + c^{1212}\frac{\partial^2 G(x^2, x^3)}{(\partial x^2)^2} + c^{1313}\frac{\partial^2 G(x^2, x^3)}{(\partial x^3)^2} + 2c^{1213}\frac{\partial^2 G(x^2, x^3)}{\partial x^2 \partial x^3}, \tag{61}$$

which may be rewritten in the form

$$c^{1111}\frac{\partial^2 F(x^1)}{(\partial x^1)^2} + \rho\omega^2 F(x^1) + c^{1212}\frac{\partial^2 G(x^2, x^3)}{(\partial x^2)^2} + c^{1313}\frac{\partial^2 G(x^2, x^3)}{(\partial x^3)^2} + 2c^{1213}\frac{\partial^2 G(x^2, x^3)}{\partial x^2 \partial x^3} + \rho\omega^2 G(x^2, x^3) = 0. \tag{62}$$

Since all terms in Eq. 62 are functions alone of x^1 or of the combination of x^2 and x^3, we may separate Eq. 62 into the following two equations:

$$\frac{c^{1111}\partial^2 F(x^1)}{(\partial x^1)^2} + \rho\omega^2 F(x^1) = -m^2 \tag{63}$$

and

$$\frac{c^{1212}\partial^2 G(x^2, x^3)}{(\partial x^2)^2} + c^{1313}\frac{\partial^2 G(x^2, x^3)}{(\partial x^3)^2} + 2c^{1213}\frac{\partial^2 G(x^2, x^3)}{\partial x^2 \partial x^3} + \rho\omega^2 G(x^2, x^3) = m^2, \tag{64}$$

where m^2 is an arbitrary constant.

Assume that $G(x^2, x^3)$ can be written in the form

$$G(x^2, x^3) = A(x^2)B(x^3). \tag{65}$$

If we substitute Eq. 65 into Eq. 64 and divide through by $G(x^2, x^3)$, we obtain

$$\frac{c^{1212}}{A(x^2)}\frac{\partial^2 A(x^2)}{(\partial x^2)^2} + \frac{c^{1313}}{B(x^3)}\frac{\partial^2 B(x^3)}{(\partial x^3)^2} + \rho\omega^2 = \frac{-2c^{1213}}{A(x^2)B(x^3)}\left[\frac{\partial A(x^2)}{\partial x^2}\frac{\partial B(x^3)}{\partial x^3} - \frac{m^2}{2c^{1213}}\right]. \quad (66)$$

The solution of Eq. 63 is

$$F(x^1) = T\sin\sqrt{\frac{\rho\omega^2}{c^{1111}}}\,x^1 + U\cos\sqrt{\frac{\rho\omega^2}{c^{1111}}}\,x^1 - \frac{m^2}{\rho\omega^2}. \quad (67)$$

Equation 66 has the general form

$$A(x) + B(y) + C = M(x)N(y) + R(x)\mathfrak{Z}(y), \quad (68)$$

where C is a constant. Differentiating with respect to x in Eq. 66, we obtain

$$\frac{\partial A(x)}{\partial x} = N(y)\frac{\partial M(x)}{\partial x} + \mathfrak{Z}(y)\frac{\partial R(x)}{\partial x}. \quad (69)$$

It is evident that in order that Eq. 69 be satisfied, either $N(y)$ and $\mathfrak{Z}(y)$ are constants or $R(x)$ and $M(x)$ are constants, or both. In other words, with the initial assumptions of the problem, only one shear motion is possible.

Let us assume

$$\frac{m^2}{B(x^3)} = k, \quad (70)$$

where k is a constant. Equation 66 reduces to

$$\frac{c^{1212}}{A(x^2)}\frac{\partial^2 A(x^2)}{(\partial x^2)^2} + \rho\omega^2 = \frac{k}{A(x^2)} \quad (71)$$

or

$$c^{1212}\frac{\partial^2 A(x^2)}{(\partial x^2)^2} + \rho\omega^2 A(x^2) = k. \quad (72)$$

The solution of Eq. 72 is similar to that of Eq. 63.

$$A(x^2) = V^1\sin\sqrt{\frac{\rho\omega^2}{c^{1212}}}\,x^2 + W^1\cos\sqrt{\frac{\rho\omega^2}{c^{1212}}}\,x^2 + \frac{k}{\rho\omega^2}. \quad (73)$$

Multiply Eq. 73 by $B(x^3) = \frac{m^2}{k}$ to obtain

$$G(x^2, x^3) = V\sin\sqrt{\frac{\rho\omega^2}{c^{1212}}}\,x^2 + W\cos\sqrt{\frac{\rho\omega^2}{c^{1212}}}\,x^2 + \frac{m^2}{\rho\omega^2}. \quad (74)$$

The displacement s^1, can now be expressed as

$$s^1 = \left[T \sin \sqrt{\frac{\rho\omega^2}{c^{1111}}}\, x^1 + U \cos \sqrt{\frac{\rho\omega^2}{c^{1111}}}\, x^1 + V \sin \sqrt{\frac{\rho\omega^2}{c^{1212}}}\, x^2 + W \cos \sqrt{\frac{\rho\omega^2}{c^{1212}}}\, x^2 \right] \sin \omega t. \quad (75)$$

To solve for frequency we must know the dimensions of the plate and the method of mounting. Let us assume the dimensions to be as follows:

length = l in the x^1 direction,
width = b in the x^3 direction,
thickness = u in the x^2 direction,

with the origin of the coordinate system located at a corner of the crystal.

Probably the most common method of mounting crystal plates is by clamping at all corners. Thus $s^1 = 0$, at each of the following points:

$(0, 0, 0)$, $(0, u, 0)$, $(l, 0, 0)$, $(l, u, 0)$,
$(l, 0, b)$, (l, u, b), $(0, 0, b)$, $(0, u, b)$.

Applying these to Eq. 75, we obtain the following equations:

$$U = -V, \quad (76)$$

$$U = \frac{T \sin \sqrt{\frac{\rho\omega^2}{c^{1111}}}\, l}{1 - \cos \sqrt{\frac{\rho\omega^2}{c^{1111}}}\, l}, \quad (77)$$

and

$$W = \frac{V \sin \sqrt{\frac{\rho\omega^2}{c^{1212}}}\, u}{1 - \cos \sqrt{\frac{\rho\omega^2}{c^{1212}}}\, u}. \quad (78)$$

As a final assumption to obtain the frequency of oscillation, consider the motion to be symmetrical about a vertical filament through the geometrical center of the rectangular crystal. That is, let

$$s^1 = 0 \quad \text{at} \quad \left(\frac{l}{2}, \frac{u}{2}, x^3 \right). \quad (79)$$

Applying Eqs. 76, 77, 78, and 79 to Eq. 75, we obtain

$$\cos \frac{u}{2} \sqrt{\frac{\rho\omega^2}{c^{1212}}} = \cos \frac{l}{2} \sqrt{\frac{\rho\omega^2}{c^{1111}}}. \quad (80)$$

From Eq. 80 we may state that

$$2n\pi \pm \frac{u}{2}\sqrt{\frac{\rho\omega^2}{c^{1212}}} = \frac{l}{2}\sqrt{\frac{\rho\omega^2}{c^{1111}}}, \tag{81}$$

where n is any integer. Thus

$$f = \frac{n}{\frac{l}{2}\sqrt{\frac{\rho}{c^{1111}}} \mp \frac{u}{2}\sqrt{\frac{\rho}{c^{1212}}}}. \tag{82}$$

It is evident then, that by careful selection of boundary conditions, Eqs. 44, 45, and 46 can be simplified to provide solutions for almost any desired modes of oscillation.

III. CONCLUSIONS.

This paper has considered a systematic tensor development of elasticity, and a statement in tensor terms of piezoelectric theory. Simple applications have been made to all of the elementary elastic deformation problems. A set of equations of motion has been set up by the combination of the elastic and piezoelectric equations previously derived, with Newton's laws of motion. Specific boundary conditions have been placed upon these differential equations to obtain solutions for displacement and frequency of natural harmonic motion of bodies with particular dimensions and specific constraining conditions.

The possibility of further applications and extensions of the work begun here is evident. The methods are the application of different and perhaps more sophisticated boundary conditions. Such development is not directly an application of the tensor theory, but falls more in the realm of the solution of partial differential equations. The quantity of empirical data being collected on these topics today is adequate evidence of its practical value.

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STRAIN ENERGY IN GREATLY DEFORMED ELASTIC OR INELASTIC ANISOTROPIC ENGINEERING METALS.

BY

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SUMMARY.

The equilibrating stresses on an *orthogonal* element in a *strained* body induce normal strains defined as *relative displacement per unit length of deformed body*. This vector "straining"-displacement must be a point function for continuity of the solid. The work done by the forces on the faces of an orthogonal element of unit volume in the loaded body during a differential increase of deformation is found. Integration gives the total work done on the element from the initial unstrained state up to the current state of deformation.

1. INTRODUCTION.

The idea of using strain energy in the study of stress-strain appears to be due to Poncelet (1, p. 534) [2] in 1830 studying the effect of impactive simple tension. Saint Venant from about 1850 gave final form to the classical equations which consider only differential displacements and strains in isotropic elastic materials and it was then a short step to utilize Poncelet's idea with the general stress-strain equations. Kelvin (2) broadened the energy conception by introducing the theorem of minimum strain energy. Saint Venant recognized the severe physical limitations imposed by the theory utilizing differentially small strains and displacements and evolved a theory (1, p. 864), but he was not successful in removing the restrictions if we can accept Todhunter's comment that the expressions apply only when the strains are small although the rates of change of displacement with position can be large (1, p. 865). The same comment will apply to Filon's large strains theory (3) which considers the co-ordinate point as referring to the deformed body, whereas in Saint Venant's method the co-ordinate point is in the unstrained body. Saint Venant's and Filon's theories lead to non-linear strain-displacement expressions so that the solution of particular elastic problems is mathematically complicated as seen in Seth's paper (19). Murnaghan in a recent paper (4) accepted Filon's expressions but then concentrated attention on the elastic strain energy function using a power series having one more term than in the classical small strain theory. Sokolnikoff states (20) that it is difficult to give a geometrical interpretation to such strains as evolved in the theories of Saint Venant, Filon and Murnaghan. For this same reason the present writer has difficulty in seeing if stress-strain compatibility has been achieved in these works.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

The expressions for energy presented in the present paper follow from the stress-strain-displacement theories due to the writer and now awaiting publication (5,6,7,8,9,10). These general ideas follow on from a theory giving stress-strain compatibility for isotropic metals yielded under small strains and displacements (11). The group of theories removes the classical restrictions by allowing the analysis of any finite displacements and strains in elastic or inelastic anisotropic engineering

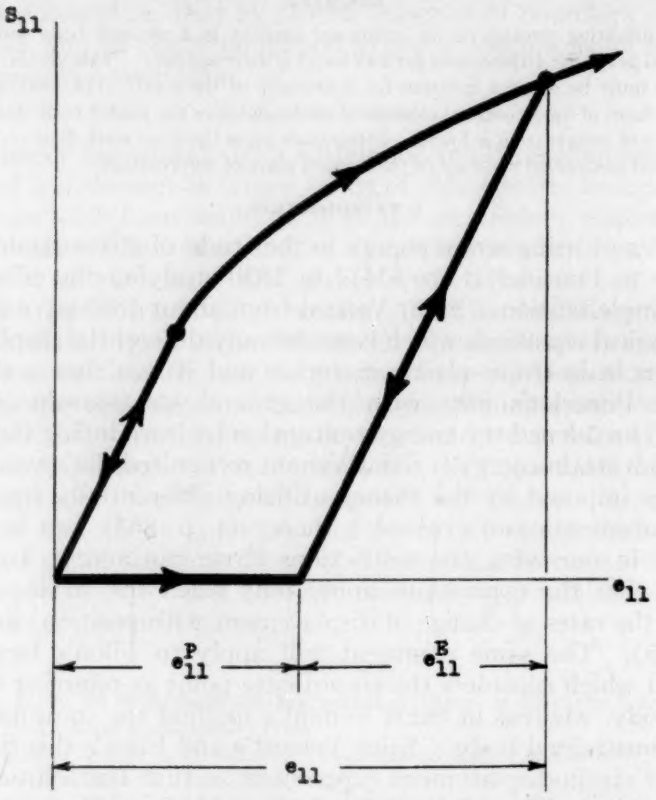

FIG. 1. Elastic and plastic components of inelastic strains in the one-stress system.

metals while still retaining linear forms in the equations. In view of delay in the publication of these theories some space has been taken up in the present paper by an abbreviated recapitulation of the ideas involved.

The degree of generality aimed at has led to more complicated notation than is usual when only the slightly strained isotropic elastic solid is considered. Vector methods were selected as most convenient to retain the physical picture and to give the simplest expressions. It is hoped the reader will be patient with the subscript notation used to ensure brevity.

2. STRESS-STRAIN COMPATIBILITY (10, 8).

Consider an *orthogonal* element in a loaded one-stress test piece. The load is applied in the direction $\mathbf{c}_1$ of an orthogonal system of unit vectors $\mathbf{c}_1$, $\mathbf{c}_2$, $\mathbf{c}_3$ and induces the "true" normal stress S_{11}.

Now define the "true" normal strain

$$e = \frac{\text{Stretched length} - \text{Initial length}}{\text{Stretched length}}. \tag{2.1}$$

Experiment shows that a strained metal is stable well into the yielded range as pointed out by Brix (1, p. 462) in 1837 and Taylor and Quinney (12) in 1932. Further, in the one-stress test the inelastic strains can be split into elastic and plastic components as in Fig. 1.

$$e_{11} = e_{11}{}^{E} + e_{11}{}^{P}. \tag{2.2}$$

Hence regard each component as accompanied by an appropriate transverse strain in the usual *static* equilibrium fashion so that the strains in the directions $\mathbf{c}_2$, $\mathbf{c}_3$ are respectively

$$\begin{aligned} e_{22,11} &= -q_{22,11}e_{11}{}^{E} - p_{22,11}e_{11}{}^{P}, \\ e_{33,11} &= -q_{33,11}e_{11}{}^{E} - p_{33,11}e_{11}{}^{P}, \end{aligned} \tag{2.3}$$

where using true strains the q's are "true" Poisson elastic transverse contraction ratios while the p's are the "true" plastic transverse contraction ratios (11,13) appropriate to the various directions to allow for anisotropy of the stress-strain parameters.

Now consider an *orthogonal* element in equilibrium under a complex true normal stress system S_{11}, S_{22}, S_{33} for which the unit reference vectors can be any orthogonal set taking the directions appropriate to the co-ordinate system to which the deformed body is referred. Suppose as the basic hypothesis for the succeeding theory that in the engineering metals considered *a normal stress is accompanied by a normal strain in the direction of the stress and by appropriate orthogonally transverse contraction effects*. Then in the absence of any experimental evidence to the contrary it is hypothesized that effects shown in (2.2) and (2.3) can be superimposed to give the total strain in direction $\mathbf{c}_i$ as

$$\begin{gathered} e_{ii} = \epsilon_{ii} + \epsilon_{jj,ii} + \epsilon_{kk,ii}, \\ i = 1, 2, 3, \qquad j = 2, 3, 1, \qquad k = 3, 1, 2, \end{gathered} \tag{2.4}$$

where ϵ_{ii} is the "partial" strain (11) that would operate in the direction $\mathbf{c}_i$ in the absence of transverse effects from the other two orthogonal directions $\mathbf{c}_j$, $\mathbf{c}_k$.

Now consider an *orthogonal* element in equilibrium under only a complex shear stress system S_{ij}, S_{ji}, where $i = 1, 2, 3, j = 2, 3, 1$. Resolve the shear stresses into components $T_{ij}/\sqrt{2}$, $-T_{ji}/\sqrt{2}$ in the direc-

tions c_{ij}, c_{ji} respectively of planes bisecting the right angle between the planes on which the shear stresses act as in Fig. 2. When these stresses are transformed into normal stresses acting on areas normal to the c_{ij}, c_{ji} then they have values T_{ij}, $-T_{ji}$ of the same arithmetical values as the shear stresses S_{ij}, S_{ji} as was first pointed out by Saint Venant (1). It is convenient to call these normal stresses T_{ij}, T_{ji} the "auxiliary" normal stresses of the shear stresses S_{ij}, S_{ji}. Compatible with the auxiliary normal stresses will be "auxiliary" normal strains e_{ij}, e_{ji} in

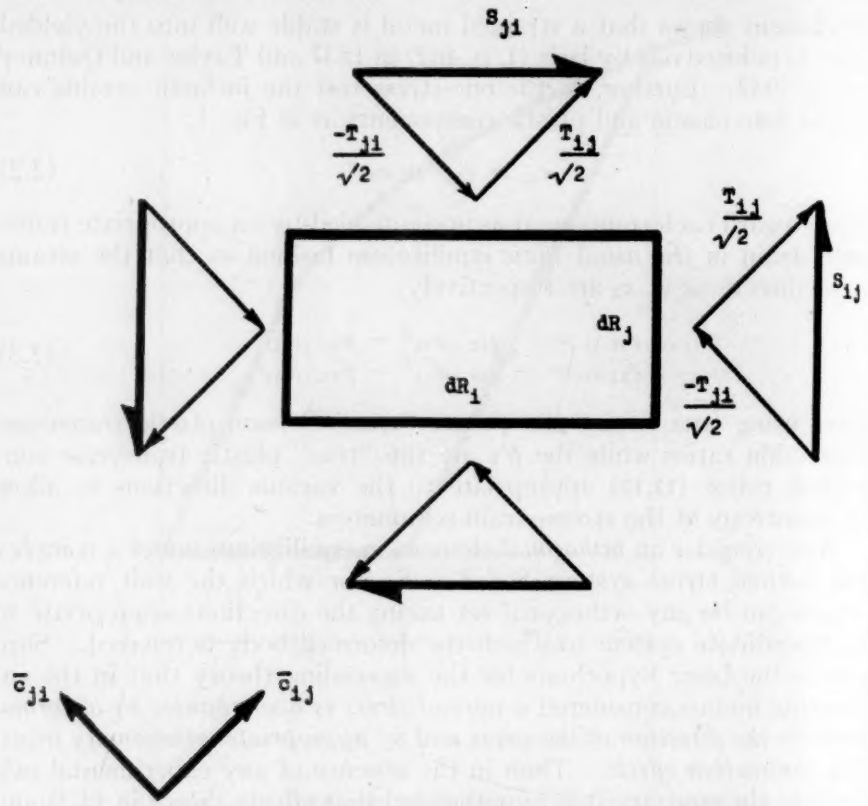

FIG. 2. Components of shear stresses acting in the directions bisecting the right angle between the planes.

the directions c_{ij}, c_{ji} respectively. Following precisely the same argument as previously we have

$$\left.\begin{aligned} e_{ij} &= \epsilon_{ij} + \epsilon_{ji,ij}, \\ e_{ji} &= \epsilon_{ji} + \epsilon_{ij,ji}, \\ e_{ij,kk} &= \epsilon_{ij,kk} + \epsilon_{ji,kk}, \end{aligned}\right\} \tag{2.5}$$

where the last term is the transverse strain in direction c_k normal to the plane of c_{ij}, c_{ji}.

If now a complex system of both normal and shear stresses operates

on an *orthogonal* element then it is hypothesized that the strains in Eqs. 2.4 and 2.5 must occur simultaneously for stress-strain compatability.

Now consider the elastic and plastic components of the inelastic strains. Suppose the progressive straining of a body is referred in general to a curvilinear co-ordinates system in which the orthogonal unit reference vectors $\mathbf{c}_i$ take local directions at each point considered. Then the current *point* of the deformed body at each instantaneous position is associated with an orthogonal element kept in equilibrium by

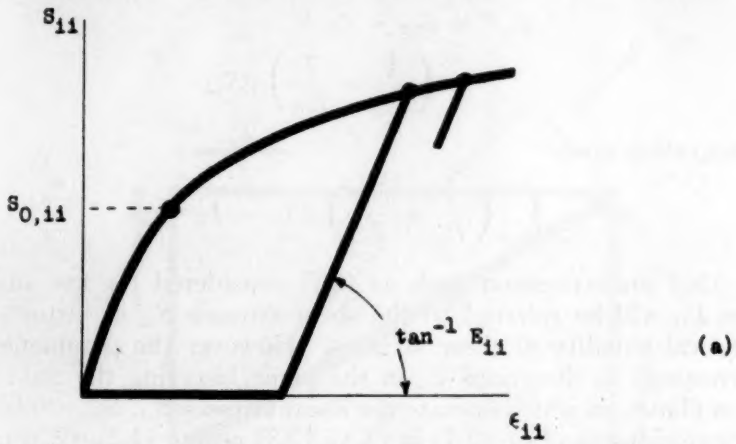

FIG. 3(*a*). Typical stress S_{ii} plotted against the corresponding "partial" strain ϵ_{ii}.

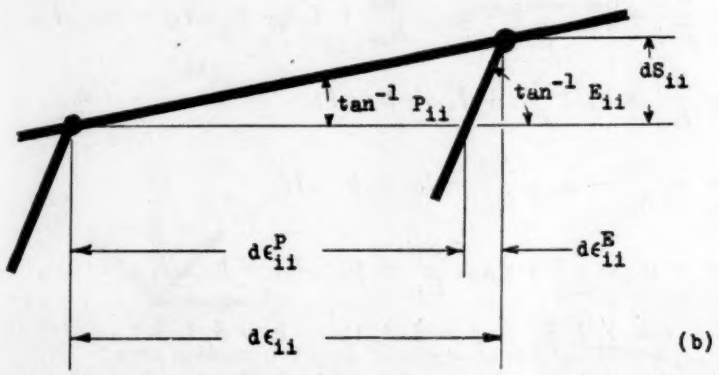

FIG. 3(*b*).

the instantaneous stresses S_{ii}, S_{ij} operating on its faces. In general *the orientation of the orthogonal element will change as the current point moves from point-to-point and if of a given size it will not always contain the same particles of the body.* As the straining proceeds the curve of the typical stress S_{ii} plotted against the corresponding "partial" strain ϵ_{ii} will have something the form shown in Fig. 3(*a*). At some instant the metal about the point will yield and the typical stress on the element will

have the value $S_{0,ii}$. From the definition of elastic strain as that part recoverable by release of load and the physical fact that Young's elastic modulus is approximately linear during this change then

$$\epsilon_{ii}{}^{E} = \frac{S_{ii}}{E_{ii}}. \tag{2.6}$$

But by definition the plastic part of the inelastic strain is the non-recoverable part with release of load. Hence considering Fig. 3(*b*)

$$\begin{aligned} d\epsilon_{ii}{}^{P} &= d\epsilon_{ii} - d\epsilon_{ii}{}^{E} \\ &= \left(\frac{1}{P_{ii}} - \frac{1}{E_{ii}}\right) dS_{ii} \end{aligned}$$

and integration gives

$$\epsilon_{ii}{}^{P} = \int_{S_{0,ii}}^{S_{ii}} \left(\frac{1}{P_{ii}} - \frac{1}{E_{ii}}\right) dS_{ii} = I_{ii} \quad \text{say.} \tag{2.7}$$

Notice that an expression such as (2.7) considered for the auxiliary stresses T_{ij} will be referred to the shear stresses S_{ij} in virtue of the arithmetical equality of these stresses. However the parameters P_{ij}, E_{ij} correspond to directions $\mathbf{c}_{ij}$ in the plane bisecting the right angle between planes on which operate the shear stresses S_{ij}, S_{ji}. Substitute such expressions as (2.6), (2.7) in (2.4), (2.5) noting (2.2), (2.3) to give

$$\left.\begin{aligned} e_{ii} &= \frac{S_{ii}}{E_{ii}} - q_{jj,ii}\frac{S_{jj}}{E_{jj}} - q_{kk,ii}\frac{S_{kk}}{E_{kk}} + I_{ii} - p_{jj,ii}I_{jj} - p_{kk,ii}I_{kk}, \\ e_{ij} &= \frac{S_{ij}}{E_{ij}} + q_{ji,ij}\frac{S_{ji}}{E_{ji}} + I_{ij} + p_{ji,ij}I_{ji}, \\ e_{ji} &= -\frac{S_{ji}}{E_{ji}} - q_{ij,ji}\frac{S_{ij}}{E_{ij}} - I_{ji} - p_{ij,ji}I_{ij}, \\ e_{ij,kk} &= -q_{ij,kk}\frac{S_{ij}}{E_{ij}} + q_{ji,kk}\frac{S_{ji}}{E_{ji}} - p_{ij,kk}I_{ij} + p_{ji,kk}I_{ji}, \\ & i = 1, 2, 3, \qquad j = 2, 3, 1, \qquad k = 3, 1, 2. \end{aligned}\right\} \tag{2.8}$$

Note that in an isotropic metal for which the tensile and compressive parameters are equal $e_{ij,kk}$ is zero and that e_{ij} and e_{ji} are equal but of opposite sign.

3. STRAIN-DISPLACEMENT COMPATIBILITY (7, 8, 9).

An *orthogonal* element in a *deformed* body is maintained in equilibrium by the complex stresses operating on its faces. The discussion of paragraph 2 indicated that in the presence of a stress there must be a strain. Thus due to the true *normal* stresses S_{ii} operating on an or-

thogonal element with edges of length dR_1, dR_2, dR_3 there must be strains e_{ii} operating. But from Eq. 2.1 defining "true" strain the differential relative "normal" displacement of the diagonally opposite corners of the orthogonal element will be

$$d\mathbf{D}^N = \sum_{i=1}^{3} e_{ii} dR_i \mathbf{c}_i. \tag{3.1}$$

The normal stresses T_{12}, T_{21} "auxiliary" to the shear stresses S_{12},

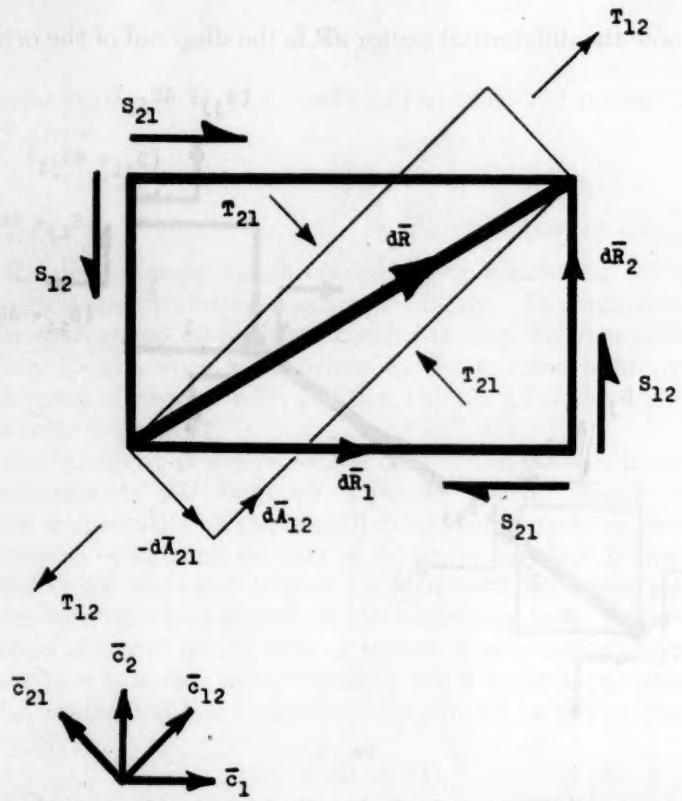

FIG. 4. Normal stresses T_{12}, T_{21}, are "auxiliary" to the shear stresses S_{12}, S_{21}.

S_{21} are shown in Fig. 4. As shown T_{12} is a tension and T_{21} is a compression. Consider the *orthogonal* element with edges of length dR_1, dR_2, dR_3. Differential position vector $d\mathbf{R}$ is parallel to the plane of $\mathbf{c}_1$, $\mathbf{c}_2$. Hence from the geometry of Fig. 4

$$d\mathbf{R}_1 + d\mathbf{R}_2 = d\mathbf{R} = d\mathbf{A}_{12} - d\mathbf{A}_{21}, \tag{3.2}$$

where the $d\mathbf{A}$'s are in the directions of the auxiliary normal stresses. Compatible with these auxiliary stresses must be auxiliary strains e_{12}, e_{21}.

From the definition of normal strains and displacements there will be a relative differential *normal* displacement between the ends of $d\mathbf{R}$,

$$e_{12}d\mathbf{A}_{12} - e_{21}d\mathbf{A}_{21} = \tfrac{1}{2}d\mathbf{R}\cdot[(e_{12} + e_{21})(\mathbf{c}_1\mathbf{c}_1 + \mathbf{c}_2\mathbf{c}_2) + (e_{12} - e_{21})(\mathbf{c}_1\mathbf{c}_2 + \mathbf{c}_2\mathbf{c}_1)] \quad (3.3)$$

after resolution to the $\mathbf{c}_1$, $\mathbf{c}_2$ directions using (3.2). Also in the direction $\mathbf{c}_3$ is a strain $e_{12,33}$ as in (2.5). Compatible with this strain will be a displacement

$$dR_3 e_{12,33}\mathbf{c}_3. \quad (3.4)$$

Hence if now the differential vector $d\mathbf{R}$ is the diagonal of the orthogonal

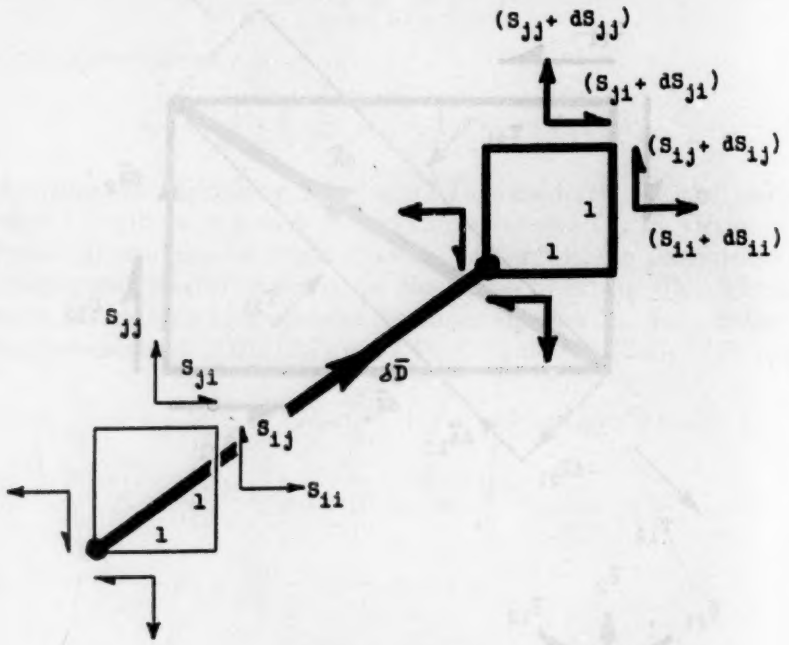

FIG. 5.

element the total differential relative displacement between its ends due to the shear stress system will be found by adding (3.3) and (3.4) and then summing for all directions to give

$$d\mathbf{D}^s = d\mathbf{R}\cdot \sum_{\substack{i=1,2,3\\ j=2,3,1\\ k=3,1,2}} \{\tfrac{1}{2}[(e_{ij} + e_{ji})(\mathbf{c}_i\mathbf{c}_i + \mathbf{c}_j\mathbf{c}_j) + (e_{ij} - e_{ji})(\mathbf{c}_i\mathbf{c}_j + \mathbf{c}_j\mathbf{c}_i)] + e_{ij,kk}\mathbf{c}_k\mathbf{c}_k\}. \quad (3.5)$$

If now a complex stress system S_{ii}, S_{ij} acts on an *orthogonal* element of a *deformed* body, then the strains compatible with the stresses will also be compatible with the straining-displacement $\mathbf{D}$. Hence by adding (3.1) to (3.5) is found the total differential relative displacement be-

tween the ends of the differential position vector $d\mathbf{R}$ diagonal to the orthogonal element

$$d\mathbf{D} = d\mathbf{D}^N + d\mathbf{D}^S. \tag{3.6}$$

But in a solid deformed body the displacement must be a point function to ensure continuity so that

$$d\mathbf{D} = d\mathbf{R}\cdot\nabla\mathbf{D}, \tag{3.7}$$

where ∇ is the vector gradient operator

$$\sum_{i=1}^{3} \mathbf{c}_i \frac{\partial}{\partial R_i}.$$

Hence substituting (3.7), (3.1) and (3.5) in (3.6) and noting that $d\mathbf{R}$ is arbitrary gives

$$\nabla\mathbf{D} = \sum_{\substack{i=1,2,3\\ j=2,3,1\\ k=3,1,2}} \{e_{ii}\mathbf{c}_i\mathbf{c}_i + e_{ij,kk}\mathbf{c}_k\mathbf{c}_k + \tfrac{1}{2}[(e_{ij} + e_{ji})(\mathbf{c}_i\mathbf{c}_i + \mathbf{c}_j\mathbf{c}_j) + (e_{ij} - e_{ji})(\mathbf{c}_i\mathbf{c}_j + \mathbf{c}_j\mathbf{c}_i)]\}. \tag{3.8}$$

This is the displacement-strain compatibility expression for any finite displacements and strains in anisotropic metals. In expansions for any particular orthogonal co-ordinate system it must be remembered that the $\mathbf{c}_i$ have local values appropriate to the current point considered and their rates of change with position calculated if the scalar expressions are to be separated.

The displacement $\mathbf{D}$ is seen as due only to the stresses inducing relative displacements $d\mathbf{D}$ between adjacent points. Thus rigid-body translation and rotation of the overall deformed body must be excluded. This is secured by causing *the axes of the strain ellipsoid at any one arbitrary point in the deformed body to coincide with the corresponding line elements of different curvilinearity in the unstrained body.* The existence of the strain ellipsoid in the case of strains of any finite magnitude as considered here is shown by considering the differential position vector $d\mathbf{R}'$ in the unstrained body corresponding to $d\mathbf{R}$ in the strained body. Geometry gives

$$d\mathbf{R}' = d\mathbf{R} - d\mathbf{D}. \tag{3.9}$$

Substitute for $d\mathbf{D}$ from (3.8) and (3.7) and then write down a similar expression using the conjugate dyadic $\mathbf{D}\nabla$. Form the scalar product of these two expressions and noting that the dyadics are self-conjugate the existence of the strain ellipsoid is proven (14, p. 94).

4. DISPLACEMENT-STRESS COMPATIBILITY.

Paragraphs 2 and 3 discussed the conditions for stress-strain and strain-displacement compatibility, respectively. Substituting the appropriate expressions gives the conditions to assure displacement-stress compatibility. This may be expressed

$$\nabla \mathbf{D} = \bar{\phi}, \tag{4.1}$$

where $\bar{\phi}$ is the dyadic found when the strain expressions (2.8) are substituted in (3.8). The dyads are of unit vectors $\mathbf{c}_i$ appropriate to the orthogonal reference system chosen and with coefficients of scalar stresses S_{ii}, S_{ij}.

An interesting result follows from taking the curl of (4.1). By a well-known result for any vector point function (14, p. 122)

$$\text{curl grad } \mathbf{D} = \nabla \times (\nabla \mathbf{D}) = 0. \tag{4.2}$$

This should be compared with Lame's analysis (14, p. 2) showing that the infinitesimal displacement in the classical theory is a biharmonic. Evidently Lame's higher order equation follows from the inclusion of rigid-body displacements whereas in the present theory they are excluded.

Again from (4.1) and (4.2)

$$\text{curl } \bar{\phi} = 0. \tag{4.3}$$

But for rigid body equilibrium of the element the stress dyadic $\bar{\psi}$ and body forces $\mathbf{F}$ obey the well-known expression (14, p. 142)

$$\text{div } \bar{\psi} + m\mathbf{F} = 0, \tag{4.4}$$

where m is the density of metal at the current point. Hence in a particular solution the stress system must obey Eqs. 4.3 and 4.4 simultaneously.

5. STRAIN ENERGY.

The analysis of the general energy case when the co-ordinate system is curvilinear follows the same lines as discussed in this paragraph for a cartesian system except that variations in the unit reference vectors $\mathbf{c}_i$ must be considered. The manipulation is much more complicated than what follows for the cartesian case and since the final expressions derived are of the same *form* except for a further integral term allowing for the way the unit vectors vary during the deformation, then it is not considered worth-while here since brevity is important. The general treatment will be given in a treatise (15) now in preparation.

A point at $\mathbf{R}$ in the deformed body has suffered the vector straining-displacement $\mathbf{D}$. A further differential displacement $\delta\mathbf{D}$ of the point is allowed. Thus Eq. 4.1 with Eq. 3.7 gives

$$d\mathbf{D} = d\mathbf{R} \cdot \bar{\phi}; \tag{5.1}$$

then due to the variation

$$d(\mathbf{D} + \delta\mathbf{D}) = d\mathbf{R} \cdot (\bar{\phi} + \delta\bar{\phi}) \tag{5.2}$$

when the co-ordinate system is cartesian and the element defined about

the current point is always of the same dimensions so that $d\mathbf{R}$ is not varied. Then (5.1) in (5.2) gives

$$d(\delta\mathbf{D}) = d\mathbf{R}\cdot\delta\bar{\phi}. \tag{5.3}$$

Now in particular to simplify the algebra and discussion, choose the orthogonal element with equal edges of unit length so that the true stresses S_{ii}, S_{ij} multiplied by unity give the *forces* operating on the faces of the element. Consider the work done by the normal force S_{ii}

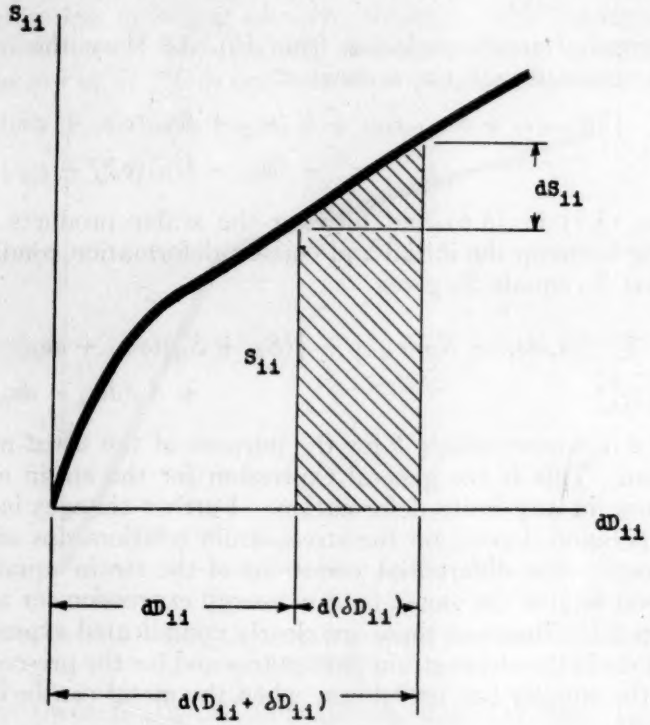

FIG. 6. Complex-stress typical direction.

operating on the face normal to $\mathbf{c}_i$ due to the variation in displacement. From Fig. 6 the work done will be

$$S_{ii}d(\delta D_{ii}).$$

Hence considering the work done on both faces normal to the $\mathbf{c}_i$ direction with the $d\mathbf{R}$ in Eq. 5.3 taking the values $\frac{1}{2}(\mathbf{c}_j + \mathbf{c}_k)$ and $(\mathbf{c}_i + \frac{1}{2}\mathbf{c}_j + \frac{1}{2}\mathbf{c}_k)$ appropriate to the faces the strain energy due to this stress is

$$dU_{ii} = S_{ii}\mathbf{c}_i\cdot\delta\bar{\phi}\cdot\mathbf{c}_i. \qquad (i = 1, 2, 3). \tag{5.4}$$

Similarly for the shear stresses

$$dU_{ij} = S_{ij}\mathbf{c}_i \cdot \delta\vec{\phi} \cdot \mathbf{c}_j,$$
$$dU_{ji} = S_{ji}\mathbf{c}_j \cdot \delta\vec{\phi} \cdot \mathbf{c}_i. \qquad (5.5)$$
$$(i = 1, 2, 3)$$
$$(j = 2, 3, 1).$$

The total differential strain energy is found by summing all such as (5.4) and (5.5) to give

$$dU = \sum_{\substack{i=1,2,3 \\ j=2,3,1}} [S_{ii}\mathbf{c}_i \cdot \delta\vec{\phi} \cdot \mathbf{c}_i + S_{ij}\mathbf{c}_i \cdot \delta\vec{\phi} \cdot \mathbf{c}_j + S_{ji}\mathbf{c}_j \cdot \delta\vec{\phi} \cdot \mathbf{c}_i]. \qquad (5.6)$$

But in terms of strain variation from Eq. 3.8 since the co-ordinate system is cartesian with the $\mathbf{c}_i$ constant,

$$\delta\vec{\phi} = \sum_{\substack{i=1,2,3 \\ j=2,3,1 \\ k=3,1,2}} [\{\delta e_{ii}\mathbf{c}_i\mathbf{c}_i + \delta e_{ij,kk}\mathbf{c}_k\mathbf{c}_k + \tfrac{1}{2}[(\delta e_{ij} + \delta e_{ji})(\mathbf{c}_i\mathbf{c}_i + \mathbf{c}_j\mathbf{c}_j) + (\delta e_{ij} - \delta e_{ji})(\mathbf{c}_i\mathbf{c}_j + \mathbf{c}_j\mathbf{c}_i)]\}. \qquad (5.7)$$

Substitute (5.7) in (5.6) and forming the scalar products and then integrating between the initial and current deformation condition after noting that S_{ij} equals S_{ji} gives

$$U = \int \sum_{\substack{i=1,2,3 \\ j=2,3,1 \\ k=3,1,2}} [S_{ii}de_{ii} + S_{kk}de_{ij,kk} + \tfrac{1}{2}(S_{ii} + S_{jj})(de_{ij} + de_{ji}) + S_{ij}(de_{ij} - de_{ji})], \qquad (5.8)$$

in which d can now replace δ for the purpose of the usual notation of integration. This is the general expression for the strain energy per unit volume in any finite deformation. Further changes in the form of the expression depend on the stress-strain relationships and parameters chosen. The differential variations of the strain equations (2.8) can be used to give the general strain energy expression for anisotropic yielded metal. However these are clearly complicated expressions due to variations in the stress-strain parameters and for the present one may consider the simpler but useful case when the metal can be considered as isotropic.

Strain Energy in an Isotropic Metal with Tension and Compression Parameters of Equal Values.

In an isotropic metal with the tension and compression parameters of equal values the auxiliary strains will be of equal values but of opposite sign in virtue of the stresses. Thus

$$de_{ij} + de_{ji} = 0,$$
$$de_{ij,kk} = 0, \qquad (5.9)$$

so that (5.8) becomes

$$U = \int \sum_{\substack{i=1,2,3 \\ j=2,3,1}} [S_{ii}de_{ii} + 2S_{ij}de_{ij}]. \qquad (5.10)$$

The variations in strains de_{ii}, de_{ij} are found from (2.8) without subscripts to the stress-strain parameters. However although the parameters are isotropic at a point they are not in general constant with change of position in the deformed body. However a useful case occurs when the parameters can be regarded as constant.

Isotropic metal with constant stress-strain parameters.

The constancy of Young's modulus E and Poisson's elastic transverse contraction ratio q is usually accepted. The constancy of the inelastic modulus P involves the assumption that yielding begins abruptly as in Fig. 7. This has been shown reasonable within engineer-

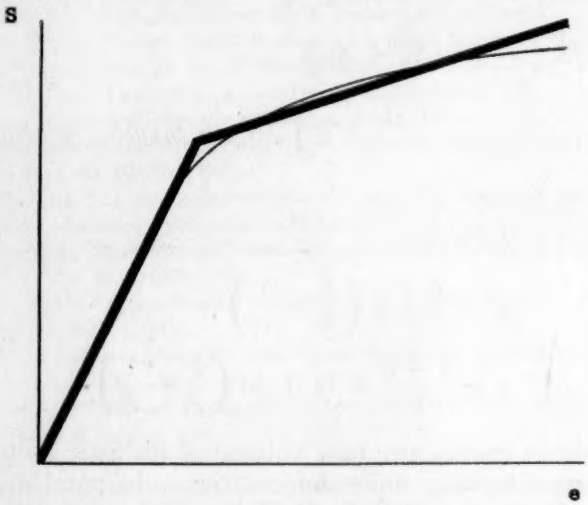

FIG. 7. Stress-strain curve showing the abrupt beginning of yield when the inelastic parameter is constant.

ing accuracy theoretically (11) and experimentally (17, 18). In reference 17 are discussed rectangular rosette readings made on a simple tensile specimen using an optical extensometer described in reference 16. The stress-strain relationships of reference 11 were used to show that even with P constant the shear stress applied to the specimen was zero. This is an elegant check of the theory because the three directions of strain measurement had to be used in the calculation and this involved the assumption of isotropy of the parameters P and p. The constancy of the plastic transverse contraction ratio p is approximately correct as shown in reference 13, once the yielding has passed the transitional range. The value of p as about 0.36 for duralumin was justified up to 40 per cent nominal strain by measurements on photogrid negatives very kindly supplied by the Armour Research Foundation, Illinois, and recently with the optical rectangular rosette extensometer up to 14 per cent nominal strain.

With these constant parameters in Eqs. 2.7, 2.8, we have

$$de_{ii} = \frac{1}{P}\,dS_{ii} - \left[\frac{q}{E} + p\left(\frac{1}{P} - \frac{1}{E}\right)\right][dS_{jj} + dS_{kk}],$$
$$de_{ij} = \left[\frac{1}{E}(1+q) + \left(\frac{1}{P} - \frac{1}{E}\right)(1+p)\right]dS_{ij}. \tag{5.11}$$

Substitute (5.11) in (5.10) and integrating by parts

$$\int S_{ii}dS_{jj} = S_{ii}S_{jj} - \int S_{jj}dS_{ii}$$

and substituting this expression gives

$$U = \sum_{\substack{i=1,2,3\\ j=2,3,1\\ k=3,1,2}} \left\{ fS_{ii}^2 - gS_{ii}(S_{jj} + S_{kk}) + g\int (S_{jj} + S_{kk})dS_{ii} + hS_{ij}^2 \right\}, \tag{5.12}$$

where

$$f = \frac{1}{2P},$$
$$g = \frac{q}{E} + p\left(\frac{1}{P} - \frac{1}{E}\right),$$
$$h = \frac{1+q}{E} + (1+p)\left(\frac{1}{P} - \frac{1}{E}\right).$$

This is the strain energy per unit volume of inelastic metal with constant parameters for any finite deformation. In particular when the metal is still elastic so that P equals E then for any finite deformation

$$U = \frac{1}{E}\sum_{\substack{i=1,2,3\\ j=2,3,1\\ k=3,1,2}} \left\{ \tfrac{1}{2}S_{ii}^2 - qS_{ii}(S_{jj} + S_{kk}) + q\int (S_{jj} + S_{kk})dS_{ii} + (1+q)S_{ij}^2 \right\}. \tag{5.13}$$

Now in particular suppose the deformation to be one of infinitesimal strains and displacements as in the classical theory so that it is plausible to hypothesize that as load increases on the structure the ratios of the stresses one to another will be constant so that for example

$$S_{jj} = X_j S_{ii},$$
$$S_{kk} = X_k S_{ii},$$

where the X's are constant, then (5.13) becomes

$$U = \frac{1}{2E}\sum_{\substack{i=1,2,3\\ j=2,3,1\\ k=3,1,2}} [S_{ii}^2 - qS_{ii}(S_{jj} + S_{kk}) + 2(1+q)S_{ij}^2]. \tag{5.14}$$

This is the classical expression for the strain energy of unit volume in an isotropic elastic body suffering infinitesimal strains and displacements as a very special case of the foregoing general results.

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NOTATION.

i, j, k used cyclically to denote three orthogonal directions at the current point of the deformed body.

$\mathbf{c}$ unit reference vector with subscripts i, j, k.

$\mathbf{R}$ position vector of the current point in the deformed body. Subscripts i, j, k indicate components.

R scalar modulus of $\mathbf{R}$.

S "true" stress. Repeated subscripts denote normal stress. Mixed subscripts denote shear stress.

T	"true" "auxiliary" stress.
e	"true" normal strain. Repeated subscripts such as ii indicate strain due to stress S_{ii} normal to the face of the element. Mixed subscripts ij denote normal strain due to shear stresses S_{ij}, S_{ji}.
E	"true" Young's elastic modulus. As an index denotes "elastic" component.
P	"true" inelastic modulus. As an index denotes "plastic" component.
q	"true" Poisson's elastic transverse contraction ratio.
p	"true" plastic transverse contraction ratio.
ϵ	"true" "partial" strain.
I	plastic "partial" strain integral.
$\mathbf{D}$	"straining"-displacement.
N, S	indices denoting normal and shear components respectively.
$\mathbf{A}$	position vector associated with a differential element in paragraph 3.
∇	vector gradient operator.
$\bar{\phi}$	dyadic first used in Eq. 4.1.
$\bar{\psi}$	stress dyadic.
m	density.
$\mathbf{F}$	body force.
U	work done per unit volume at the current point during the deformation.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

PARTICLE-SIZE DISTRIBUTION IN POWDER METALLURGY.

Advances in the science of powder metallurgy—the art of making objects by pressing and heating metal powders—have long been hindered by the difficulty of obtaining reproducible results in large-scale manufacture. This is due principally to present methods and techniques which introduce numerous variables, as yet not completely understood or controllable. In conjunction with the current program of standardization of test methods and techniques in this field, the National Bureau of Standards undertook an extensive investigation of the conditions contributing to the lack of reproducibility in sieve analyses of metal powders. Such analyses of particle-size distribution are of major importance in powder metallurgy.

Investigations at the Bureau have revealed that atmospheric humidity has a marked effect on the results obtained by sieve analyses of metal powders and that controlled atmospheric conditions during sieve testing of metal powders may, therefore, be necessary when close control of particle size is desired. Increase in humidity tends to increase the weight of the fractions retained on the sieves and decrease the weight of the pan fraction. Differences of as much as 10 per cent between the weight of fractions of powdered iron sieved under high and low humidities have been observed.

In sieve tests of sponge iron, electrolytic iron, electrolytic copper, and nickel, made for the purpose of accumulating supplies of sieved fractions of these powders for other studies, it was found that reproducible results could be obtained only when certain variables were controlled. Significant differences in sieve analysis often were obtained when samples of the sieve powder were sieved at different times with the same sieves. Furthermore, different sets of certified sieves used for the same powder gave variations of considerable magnitude. A contributing factor, in addition to atmospheric humidity, was a cumulative sampling error that resulted from repeated riffle cutting of limited powder supplies.

The effects of these variables were demonstrated by tests on sponge iron. This powder was made from reduced mill scale and consisted of irregular platelike particles. Many of the larger particles were made up of several such plates held together by the oxide of the metal.

* Communicated by the Director.

The sampling procedure included the use of a riffle-type sample splitter to reduce the entire supply of metal powder (50 to 100 lb.) to "sample supplies" which could be stored in one to five 1-pt. Mason jars (3 to 15 lb.).

Bureau tests of the effect of high humidity on the sieving characteristics of sponge iron illustrated the difficulty of reproducible analyses of metal powders. The set of certified U. S. Standard sieves used in these tests, Sieve set 3, included Nos. 100, 140, 200, 230, and 235. One hundred-gram samples taken from the same freshly riffle cut sample supply were sieved for periods of 5, 10, 15, 20, 30, 40, and 60 min. after the following treatments of the powder:

TABLE I.

Sieve tests of sponge iron showing variations due to replacement of sieves.

Average of three tests on 100-g. samples sieved 30 min. with certified sieves.

Sieve No.	With original No. 325 sieve		With new No. 325 sieve (NBS No. 8887)		Difference from original (3) minus (1)	With new No. 325 sieve (NBS No. 8888)		Difference from original (6) minus (1)	Difference between new sieves (6) minus (3)
	Mean	Deviation from mean	Mean	Deviation from mean		Mean	Deviation from mean		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Percentage, by weight, of sponge iron retained with sieve set 1								
80	Trace		Trace			Trace			
100	Trace		Trace			Trace			
140	7.3	+0.1 to 0.2	7.0	+0.1 to 0.2	−0.3	6.9	+0.1 to 0	−0.4	
200	22.6	± .1	22.4	+ .2 to .4	− .2	22.6	± .1	0	
325	31.2	+ .4 to .5	38.2	± .1	+7.0	41.0	+ .4 to .2	+9.8	+2.8
Pan	38.4	+ .2 to .1	32.0	+ .2 to .1	−6.4	28.9	± .3	−9.5	−3.1
	Percentage, by weight, of sponge iron retained with sieve set 2								
80	Trace		Trace			Trace			
100	Trace		Trace			Trace			
140	4.4	±0.6	5.2	+0.1 to 0	+0.8	5.1	0	+0.7	
200	24.4	± .6	23.5	+ .1 to 0.3	− .9	23.3	±0.1	−1.1	
325	33.0	± .2	38.6	± .3 to .2	+5.6	40.8	+ .1 to 0	+7.8	+2.2
Pan	37.9	± .2	32.2	± .1	−4.7	30.2	± .1	−7.7	−2.2

Series III.—For each time period three samples were oven dried for 1 hr. at 110°C. prior to sieving.

Series IV.—For each time period three samples were exposed for 64 to 72 hr. to a humid atmosphere in a closed vessel (desiccator) over water, with wicks dipped into the water to increase the evaporating surface.

Series V.—After sieving, the humidified samples of Series IV were mixed, dried for 5 hr. at 110°C., and resieved.

For the humidified samples, Series IV, the amounts of material retained on each sieve were consistently lower than those of the dried material, Series III. For example, differences in the case of the pan fractions ranged from 3.5 to 6.5 per cent of the original weight of the sample. Part of this difference (approximately 25 per cent) was re-

covered when the humidified samples were dried and resieved, as shown by the values obtained for Series V. Values for the individual samples of Series III lay close to the curve of the plotted results, while the values for Series IV and V were more scattered. Similar but smaller differences were obtained for the other sieve fractions.

Two additional series of tests were made using the same sieves and the same procedure as in Series III, but with samples taken from different sample supplies. The differences between each succeeding series, in the order in which they were riffle-cut, amount to about 1 per cent of the original weight of the sample. It is believed that these differences were due chiefly to the loss of fines as dust during riffle cutting.

TABLE II.

Average opening of sieves as measured in certification tests.

Sieve No.	NBS Certification No.	Average opening, microns*	
		Between warp wires	Between short wires
		Original sieve set I	
80	8805	186	174
100	8806	143	148
140	8807	107	102
200	8808	78	75
325	8809	46	46
		Original sieve set II	
80	8889	185	174
100	8890	144	140
140	8891	104	107
200	8892	77	73
325	8893	43	45
		New No. 325 sieves	
325	8887	41	43
325	8888	41	42

* Measurements were made by the Bureau's Metrology Division with an accuracy sufficient to determine whether the openings were within the limits permitted by specification. The errors are probably not in excess of about 1 micron.

Variations of considerable magnitude also were noted when powders were sieved with different sets of certified sieves. The results obtained with sponge iron when replacements of the No. 325 sieve were made in two sieve sets are given in Table I. The tests were made under approximately the same sieving conditions and all samples were taken from the same sample supply.

The fractions retained by the sieves which were not replaced (all sieves larger than No. 325) agreed closely. The new No. 325 sieves retained 7.0 and 9.8 per cent more material, respectively, with sieve set 1; and with sieve set 2, 5.6 and 7.8 per cent. The amounts retained by the two new sieves differed by 2.8 per cent when used with sieve set 1 and 2.2 per cent with sieve set 2.

In connection with these differences it is interesting to compare the

average openings of the several sieves as measured during the certification tests. These measurements are given in Table II. From a plot of the values of Table I against the measurements of the average openings, it is evident that the differences between sieves are considerably less than they would be if the comparison were made on the basis of the nominal opening. The agreement probably would be even closer if the variations in sieve openings could be taken into consideration. Due to manufacturing limitations the dimension tolerences of wire cloth permitted by specifications are necessarily rather wide, particularly for fine sieves. These variations are not fully reflected by the measurement of the average opening.

Further investigations of possible methods of eliminating, controlling, or evaluating the effects of these and other variables encountered in sieve analyses of metal powders are needed, and studies toward this end are under way at the National Bureau of Standards.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, WEDNESDAY, MAY 19, 1948.

The Stated Monthly Meeting of The Franklin Institute was held on May 19, 1948 in the Lecture Hall of the Institute. The President, Mr. Richard T. Nalle, called the meeting to order at 8:15 p.m. There were approximately 225 persons in attendance.

The President stated that the minutes of the monthly meeting for March had been printed in full in the April issue of the JOURNAL OF THE FRANKLIN INSTITUTE, and if there were no corrections or additions the minutes would stand approved as printed. There was no dissent.

The President then presented an abstract of the annual report of the Board of Managers. He gave a brief resumé of the outstanding events and the progress made by The Franklin Institute in the year 1947. He further stated that the complete annual report would be printed in full in a subsequent issue of the JOURNAL for the benefit of the members of The Franklin Institute.

The report of membership for The Franklin Institute for the month of April was presented as:

Active	26
Associate	16
Student	8

The total Institute membership as of April 30, 1948 was 5329.

Our President then spoke of the Charles Day Lecture Foundation, which he said was established as a memorial to Mr. Charles Day, who was an outstanding industrialist and a distinguished mechanical engineer. He was a lecturer and author of many technical papers on industrial problems of scientific management, and a good friend of The Franklin Institute. Mr. Day was a valued member of the Board of Managers from 1908–1931, and a Vice President from 1920–1923.

The speaker of the evening, Mr. William J. Meinel, President of Heintz Manufacturing Company was then called upon to speak. Mr. Meinel chose for his topic "The Full Responsibility of Management." This paper will be printed in full in the July issue of the JOURNAL. After Mr. Meinel's talk, a very edifying and enlightening session was held, wherein members of the audience questioned Mr. Meinel on various aspects of management. Mr. Meinel's answers were most helpful to everyone.

The meeting was adjourned at 9:45 p.m., with a rising vote of thanks to the speaker of the evening.

HENRY B. ALLEN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, May 12, 1948.)

HALL OF THE COMMITTEE,
PHILADELPHIA, MAY 12, 1948.

DR. JULIAN W. HILL in the Chair.

The following report was presented for final action:

No. 3194: Ballantine Medal.

This report recommended the award of the Stuart Ballantine Medal to Ray Davis Kell, of Princeton, New Jersey, "In consideration of his outstanding pioneer work in Television;

the adaptation of this means of communication to military needs and for his inventive contributions and leadership in the development of Color Television."

JOHN FRAZER,
Secretary to Committee.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

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Theory of Propellers, by Theodore Theodorsen. 164 pages, 16 × 23 cm., drawings, tables, and charts. New York, McGraw-Hill Book Co., 1948. Price, $6.00.

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The Genius of Industrial Research, by D. H. Killeffer. 263 pages, 15 × 23 cm., drawings. New York, Reinhold Publishing Corp., 1948. Price, $4.50.

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The Radio Amateur's Beam Pointer Guide, by John F. Rider. 30 pages, 21 × 28 cm., illustrations. New York, John F. Rider Publisher, Inc., 1948. Price, $1.00 (Paper).

Preparation and Characteristics of Solid Luminescent Materials, Symposium held at Cornell University, October 24–26, 1946, sponsored by The Division of Electron Optics of The American Physical Society, published under the auspices of The National Research Council. 459 pages, 15 × 22 cm., drawings and illustrations. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $5.00.

BOOK REVIEWS.

ENERGY UNLIMITED. THE ELECTRON AND ATOM IN EVERYDAY LIFE, by Harry M. Davis. 273 pages, plates, 14 × 22 cm. New York, Murray Hill Books, Inc., 1947. Price, $4.00.

Mr. Davis, a newspaper and magazine writer on scientific matters, has brought together in this book some of the articles which he wrote for the *New York Times Sunday Magazine* and *Popular Science Monthly*. These contribute to ten of the seventeen chapters. Writing lucidly and interestingly for the average reader, Mr. Davis discloses the exciting panorama that is the result of man's new discoveries in the field of nuclear physics and electronics. Technicalities are held to the minimum, and the definitions of terms that cannot be avoided are often repeated throughout the text.

In the first five chapters the history and elementary background of the more recent advances in physicial science are given. Chapters Six to Ten deal in more detail with the application of modern electronics, while Chapters Eleven and Twelve show the relationship of the electronic and nuclear fields, the latter being then taken up in the remaining five chapters.

The treatment of the material in this book is comprehensive to the extent that the reader, whether he be equipped with a varying degree of technical background or not, will gain a unified outlook on the achievements and problems of physical sciences that face the world at large today.

HENRY N. MICHAEL.

NEUTRON EFFECTS ON ANIMALS, by the Staff of the Biochemical Research Foundation. 198 pages, illustrations, 15 × 23 cm. Baltimore, The Williams & Wilkins Co., 1947. Price, $3.00.

With the atomic age upon us, knowledge of the effects of atomic energy becomes important. The worker in atomic energy plants needs to be protected from radiation, and, if he should be exposed, he must receive adequate remedial and curative measures. The Biochemical Research

Foundation, which for fifteen years has been studying various radiation effects, has lately been investigating the radiation effect of the neutron on cells.

The present volume, essentially preliminary in nature, is a report of these experiments on the effects of neutrons on animals and has been published to permit others to secure a general perspective of the problem.

Dr. McDonald offers first a general statement concerning radiation effects on cells. In this he points out that while neutron bombardment and X-radiation may produce similar results, there is no proof that they cause these results by a similar mechanism. Also "the most significant fact in neutron bombardment is its effect upon growing or immature tissue." Once the mechanism of this action is understood, the possibility arises that similar conditions may be produced in cancer cells, which are relatively immature, rapidly-dividing cells. The hope of thus producing a stoppage of cancer cell growth is an alluring goal.

The second chapter compares the physical properties of fast neutrons with other types of radiation while the third chapter describes the general techniques employed in the neutron irradiation procedure. The subsequent chapters are devoted to individual reports on the different experiments. Various subjects were used including rats, chickens, rabbits, dogs, corn and certain bacteria. The scope of the work is best indicated by some chapter titles: Relation between Neutron Dose and the Mortality, Body Weight, and Hematology of White Rats; Effects of Neutrons on Early Root Development of Zea Mays; Studies of Proteolytic Enzymes in Bone Marrow; Electrophoresis of the Plasma of Dogs Irradiated with Neutrons.

This is a report of significant work in a new field, which should prove a stimulus to further study. It is gratifying to know, as a prefatory note states, that considerable progress has already been made in explaining the *mechanism of radiation* on mammalian cells, findings which it was not possible to insert in the present volume.

G. E. PETTENGILL.

COMPOSITION AND PICTURES, by Eleanor Parke Custis. 224 pages, illustrations, 22 × 29 cm. Boston, American Photographic Publishing Co., 1947. Price, $6.00.

A photographer or an artist may be truly artistic and may have a natural instinct for making pictures. But no matter how strong this instinct is, it is more workable when backed by actual knowledge. In her book Miss Custis aims at presenting the fundamental rules of composition which will enable the artist to govern his perhaps undisciplined instincts. The language of the book is simple and direct, and the points of discussion are profusely illustrated with the author's own work and with that of many others.

HENRY N. MICHAEL.

UNDERSTANDING VECTORS AND PHASE, by John F. Rider and Seymour D. Uslan. 153 pages, illustrations, 13 × 19 cm. New York, John F. Rider Publisher, Inc., 1947. Price, $0.99 (paper).

This short text is intended for the man who has not had an engineering background, yet is thrown in contact with both vectors and phase in his efforts to keep abreast of electronic developments. This means the radio serviceman, the radio technician, the radio amateur.

Those who have a strong desire to know more about the field in which they are engaged, and who have a general appreciation of radio principles, will find the information contained in this booklet of much value to their understanding of articles in radio engineering publications and textbooks.

HENRY N. MICHAEL.

FACTUAL COMMUNICATION: A HANDBOOK OF AMERICAN ENGLISH, by L. O. Guthrie. 448 pages, illustrations, 15 × 22 cm. New York, Macmillan Co., 1948. Price, $3.50.

In this textbook on American English, the author has stressed the practical uses of English in articles, talks, business letters, and reports. The first part deals with different types of spoken and written communication. To guide the student further in his composition, the second part introduces pertinent theories and suggests procedure for finding information. A comprehensive section on grammar and rules forms the concluding part entitled "Handbook."

Since most of the examples use scientific and technical vocabulary, this text will be especially useful with students in technical institutions.

G. E. PETTENGILL.

FUNDAMENTALS OF PHOTOGRAPHIC THEORY, by T. H. James and George C. Higgins. 286 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1948. Price, $3.50.

This book gives a general account of the theory of the photographic process, based on the fundamental chemical and physical concepts. Thus a specialist's knowledge of chemistry and physics is not required, although their basic knowledge on the part of the reader is presupposed.

Only the black-and-white photographic processes which involve the use of silver salts are considered here. The field of color photography is not treated specifically, since every major color process involves primarily a black-and-white process.

The comprehensive nature of the book, in so far as black-and-white photography is concerned, should make it appealing both to the intelligent amateur and to the specialist.

HENRY N. MICHAEL.

INTRODUCTION TO MODERN PHYSICS, by F. K. Richtmyer and E. H. Kennard. Fourth edition, 759 pages, illustrations, 16 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1947. Price, $6.00.

The fourth edition of this extensively used text-book preserves the aims of the first edition of 1928, namely, "to present such a discussion of the origin, development, and present status of some of the more important concepts of physics, as will give the student the correct perspective of the growth and present trend of physics as a whole." In pursuing this aim the author has not given a collected bibliography, nor a list of problems and suggested topics for study. Instead, references, in most cases to original sources, have been given at the appropriate points in the text.

After the death of Dr. Richtmyer, Dr. Kennard assumed the task of revising the book. Thus the third edition of 1942 presented major changes. The historical introduction was abbreviated. The chapters on electromagnetism, the nuclear atom, and the X-rays were rewritten and new chapters on relativity and on cosmic ray phenomena inserted.

The present edition (1947) records the more significant advances of the last five years. It also reflects on further changes in perspective on the physical scene.

The notable changes are evidenced in Chapters XI (The Nucleus) and XII (Cosmic Rays), where extensive additions have been made. Chapter VIII, on many-electron atoms, has been rewritten and simplified. The entire discussion proceeds in terms of quantum states rather than of wave functions.

HENRY N. MICHAEL.

MODERN TIMBER DESIGN, by Howard J. Hansen. Second edition, 312 pages, illustrations, 15 × 23 cm. New York, John Wiley and Sons, Inc., 1948. Price, $4.50.

The second edition of "Modern Timber Design" is brought up-to-date to conform with the specifications for working stresses which were adopted since the first publication in 1943. These specifications increased the working stresses by 20 per cent. Thus the revision is quite extensive, since the above-mentioned modification affected the material in practically every chapter. Also, some new material not found in the first edition was added. This new material deals with the testing methods for wood and factors affecting the strength of wood.

The text is divided among eleven chapters. A list of headings will indicate its comprehensiveness: I. Characteristics and Properties of Wood; II. Mechanical and Related Properties; III. Grading Rules and Working Stresses; IV. Mechanics of Wood; V. Fastenings; VI. Beams and Columns; VII. Timber Decks and Bridges; IX. Glued Laminated Construction; X. Plywood; and XI. Decay, Wood-Destroying Organisms, and Preservatives.

The Appendix, among others, contains a list of definitions of terms used in describing standard grades for lumber.

Since this text-book also contains all the basic information needed by the designing engineer, it recommends itself as a guide and reference for practicing engineers.

HENRY N. MICHAEL.

PRINCIPLES OF FOOD FREEZING, by Willis A. Gortner and others. 281 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1948. Price, $3.75

The treatment of foods prior to freezing, the care and operation of the freezing equipment, the temperature of storage, and the thawing and cooking procedures used in the kitchen are all important factors in determining the nutritive value of the product as it reaches the table. The proper procedures are not difficult but the extent to which they are understood and applied will determine the success of the consumer experiments now being made. This timely book deals with all the aspects involved—from the raw materials to the finished product. It is technical in the sense that it explains the biochemistry and nutrition in freezing, storage, and cooking processes, and deals with engineering principles involved in the construction and operation of the equipment. In this connection numerous references are given. The food packer and locker operator will profit from this information. However, the book should prove of equal value to the intelligent homemaker, who although not interested in the chemical or engineering bases, may want to be able to select the proper equipment and carry out the cooking processes which will insure the best results. For this latter use the information is presented in easily understandable language.

HENRY N. MICHAEL.

PROBLÈME GÉNÉRAL DE LA STABILITÉ DU MOUVEMENT, by M. A. Liapounoff. Pages 203–474. 23 × 15 cm. Princeton, Princeton University Press, 1947. Price, $3.50.

This publication, which forms Number 17 of the *Annals of Mathematics Studies*, is a photolithoprint reproduction from the *Annales de la Faculté des Sciences de Toulouse*, Second Series, Vol. 9 (1907). This latter was itself a translation from the original Russian issued in 1892 by the Societé Mathématique de Kharkow. The author treats of methods employed in investigations of the properties of movement and of equilibrium, known as stability and instability. Two different cases are considered, one for constant movements and the other for periodic.

The *Annals* is performing a valuable service in making this study more widely available.

G. E. PETTENGILL.

MATHEMATICAL TABLE MAKERS, PORTRAITS, PAINTINGS, BUSTS, MONUMENTS, BIO-BIBLIOGRAPHICAL NOTES, by Raymond Clare Archibald. 82 pages, portraits, 17 × 24 cm. New York, Scripta Mathematica, 1948. Price, $2.00.

In the daily use of mathematical tables, probably few give much thought to the men who made these tables. This slender volume should stimulate such an interest for it is a helpful guide to the portraiture and literature of fifty-three mathematical table makers. The entry for each individual gives his full name, dates, and a brief biographical note. This is followed by a list of references to original portraits, busts, monuments, or to reproductions of them. Lists of biographical material not accompanied by illustrations, and lists of published tables, are also given.

Portraits are included for twenty of the men discussed, including most of the living ones. The text, reprinted from *Scripta Mathematica* in a revised and enlarged version, will prove of even more value in this form.

G. E. PETTENGILL.

THE CORROSION HANDBOOK, edited by Herbert H. Uhlig. 1188 pages, illustrations, tables, diagrams, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1948. Price, $12.00.

Sponsored by the Electrochemical Society and written by 103 contributors, including many of the outstanding figures in corrosion research, this volume will undoubtedly rank as one of the most important technical books of the year. It brings between two covers a summary of

corrosion information covering all fields and representing a cross section of scientific data and industrial experience.

In compiling the volume, emphasis has been placed on quantitative information. Actual corrosion rates have been given, with exposure conditions defined as precisely as possible. To facilitate comparisons and to establish standard units, most data have been transposed to inches penetration per year (ipy) or milligrams per square decimeter per day (mdd).

After a brief section on corrosion theory, the corrosion of the various metals and alloys in liquid media, in the atmosphere and in gases is discussed. This includes under each metal such topics as acids, alkalies, salt solutions, velocity, effect of mechanical factors (stress and vibration), organic compounds, and atmospheric corrosion. Although not strictly corrosion, the deterioration of various nonmetallic materials and semi-metals is also treated, because of their frequent use under similar conditions.

In the third section special topics are considered such as corrosion by sea water, by soils, by lubricants, and the effects of mechanical factors on corrosion. Section four deals with high-temperature corrosion of the more important metals with the addition of carbon and graphite, and refractories. The next two sections present tables of high-temperature and chemical-resistant materials. The latter appears of great practical value for it evaluates the resistance of twenty-five groups of materials to twelve different groups of chemicals. The materials are rated in three classes in accordance with their suitability for use under varying conditions.

The problem of corrosion protection is divided into three major parts: metallic coatings, inorganic coatings and organic coatings. Corrosion testing, both in the laboratory and in the field, is the subject of the eighth section. Miscellaneous information and general tables are gathered in a concluding section.

Numerous tables and diagrams and ample reference to the literature on corrosion are features of the book. It should be noted, too, that many data previously unpublished have also been included. A comprehensive index insures ready access to the contents.

The volume goes a long way toward making readily available the present knowledge of corrosion. It also reveals gaps in that knowledge, which, it is to be hoped, will stimulate further experimental work. In view of the high cost of depreciation and the ever-increasing necessity of preserving our raw materials, this volume should be readily available to anyone working with metals where corrosion is a factor.

G. E. PETTENGILL.

DETOXICATION MECHANISMS. THE METABOLISM OF DRUGS AND ALLIED ORGANIC COMPOUNDS, by R. Tecwyn Williams. 288 pages, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1948. Price, $5.50.

Present knowledge of the metabolic fate of even simple organic molecules is often incomplete. However, since in recent years synthetic organic compounds have been used increasingly in the treatment of various diseases, what happens to these compounds in the body has become an important question. The object of this book is to present the available material, so that a working hypothesis may be advanced.

In the past most published material concerned itself with qualitative analysis, without reference to the quantitative aspects. It is probable that in such cases other undetected metabolites have formed.

Dr. Williams' treatise covers the metabolism of the following compounds: aliphatic compounds and cyclohexane derivatives; aromatic hydrocarbons; halogenated aromatic hydrocarbons; phenols; aromatic alcohols, ethers, aldehydes, ketones, and amides; aromatic acids; organic cyanides; aromatic nitro, amino, and azo compounds; sulphones, and sulphonic acids, and sulphonamides; terpenes and camphors; heterocyclic compounds; and organic compounds of arsenic. The concluding chapter deals with the hypotheses prevalent in the past and formulates the criteria upon which a working, probably non-unitary, hypothesis must be based.

This book should prove of high value to the student and particularly to the research man who is interested in the behavior of drugs in the animal body.

HENRY N. MICHAEL.

STRESS ANALYSIS AND DESIGN OF ELEMENTARY STRUCTURES, by James H. Cissel. Second edition, 419 pages, illustrations, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1948. Price, $5.00.

This textbook presents the basic principles and procedures related to stress analysis and design of simple structure. It is intended for use in those courses of study in engineering other than civil, which include a section on structures. Thus, fundamental, useful and practical material was selected, to provide a self-contained working book which eliminates the need for auxiliary handbooks.

The 1948 edition retains in Part I essentially the same material with the exception of Chapter 2, where up-to-date information on live loads is given. However, Part II, which covers the design of simple structures, is completely rewritten and is consistent with the most recent specifications for designing structures. A new chapter on light-gage steel construction is added. Each chapter offers data for problems as an aid to class instruction.

HENRY N. MICHAEL.

HYDRAULICS, by Horace W. King and others. Fifth edition, 351 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1948. Price, $4.00.

Twenty-six years have elapsed since this book was first published. Then, as now, it was intended as an elementary text for the engineering student. In the 1948 revision the material is rearranged and clarified, and recent accepted developments, both theoretical and practical, are integrated. This new material deals with the variation of hydrostatic pressure with altitude in a compressible fluid, flow through gates and over dams, flow of liquids in pipes and pipe networks, and resistance offered to motion of objects through fluids. An entirely new chapter on hydraulic similitude and dimensional analysis covers the fundamentals upon which the increasingly important field of hydraulic model testing is based.

The unchanged popularity of this textbook throughout a quarter century testifies to its thoroughness and usefulness.

HENRY N. MICHAEL.

STEEL AND ITS HEAT TREATMENT, by D. K. Bullens. Fifth edition, Vols. I and II, 489 and 293 pages, respectively, illustrations, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1948. Price: Vol. I, $6.00; Vol. II, $4.00.

The fifth edition of this work, which first appeared in 1915, continues in its principles of combining theory and practice for use in conjunction with observation of steel behavior.

Since 1938, the date of the fourth edition of this work, many new methods in the treatment of steel were developed, particularly during the war. These developments, hitherto published fragmentarily, are sifted, and the especially illuminating ones are gathered in an arrangement which aids the practical heat treater.

The present edition is to appear in three volumes, two of which have already been published and are considered here.

Volume I covers the definitions, terminology and fundamental concepts underlying the discussion of heat treatment. It points out the limitations and uses of tests applied to determine suitability of steel for heat treatment, and the suitability of the end-product for engineering use. The principles governing the various types of heat treatment, both of carbon and alloy steel, are thoroughly investigated.

In Vol. II these principles are applied and the tools of heat treatment are described. These are the furnaces, salt baths, controlled atmospheres, quenching media, etc. It also covers such items as prevention of cracking and distortion, conditioning of steel for machining, welding, and the heat treatment of steel castings, malleable iron, and cast iron.

H. N. MICHAEL.

INVENTIONS, PATENTS AND MONOPOLY, by Peter Meinhardt. 352 pages, 14 × 22 cm. London, Stevens & Sons, Ltd., 1946. Price, 25 s.

The author states that Great Britain was the first country in the world where patents

were issued to inventors. The British Patent System undoubtedly has contributed to making the country one of the greatest industrial nations and the law has been adopted with modifications in more than a hundred other countries. In the latter respect it is a part of English law carried with British colonists and used as a basis in the newly settled countries. This volume, written and produced in Great Britain, is primarily given to that country. The larger part of it is devoted to British patent law and practice. This part is mainly descriptive and quite simple, outlining the requirements facing one applying for a patent and the patentee who wants to enforce his rights against an infringer. Other sections in this part deal with the maintenance of the patent, tax liabilities, and patent litigation. Of great general interest to those outside of Great Britain are other parts of the book. The first part dealing with the characteristics of inventors and inventions is in this class and is quite novel. Circumstances, psychology, economics, practical matters, rewards, motives, etc., are all touched in a very human way. In another part of the book, the abuse of the patent monopoly is a topic of a more general interest and a timely subject here in the United States as well as in Great Britain. Charging excessive prices for patented articles, and prejudicing trade or industry by the refusal to grant licenses are given consideration. The last part of the book on suggestions for patent law reform again refers to the British situation.

The book will be of interest to all persons, here and abroad, who desire a knowledge and understanding of inventions and patents.

R. H. OPPERMANN.

BROADCAST OPERATORS HANDBOOK, by Harold E. Ennes. 265 pages, illustrations, 14 × 21 cm. New York, John F. Rider Publisher, Inc., 1947. Price, $3.30.

Although there are numerous books on radio equipment and several on program arrangement, there has been little written on the practical operation of broadcast equipment to insure successful transmission of programs. This volume by the staff engineer of station WIRE aims to fill this gap in the literature. The first four parts are devoted to operating practice in the control room, the master control, remote controls and the transmitter. Among the many points touched on are control of loudness, operation of turntables, location of the microphone for various types of performances, and pick-ups of different programs outside the studio.

The fifth part is devoted to emergency shut-downs with a suggested program of preventive maintenance. This includes both a comprehensive schedule of points to check and instructions on how to effect the maintenance carefully and adequately. The author differentiates six types of operation—feel, inspect, tighten, clean, adjust and lubricate—and discusses their application to the various types of equipment. A final part mentions some of the technical equipment used and offers suggestions on developing a broadcast studio and selecting broadcaster transmitter location.

A practical book by a practicing engineer, this volume should prove helpful to the newcomer in broadcast station operation, as well as a source of valuable hints for the more experienced man.

G. E. PETTENGILL.

JET PROPULSION PROGRESS. THE DEVELOPMENT OF AIRCRAFT GAS TURBINES, by Leslie E. Neville and Nathaniel F. Silsbee. 232 pages, illustrations, 15 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1948. Price, $3.50.

A major revolution in aviation is taking place with the introduction of gas turbines in jet-propelled airplanes. In the present volume the authors have given an outline of the principal developments in jet propulsion, though there has been no attempt to write a complete history, or to mention many designs, patents and experimental projects. As the authors point out, this latter has been well done by G. Geoffrey Smith.

After a brief introductory chapter on the different types of gas turbines, the work of the Germans is reviewed. The jet-propelled plane never played a decisive role in World War II, since the Messerschmitt Me-262 was the only jet plane which was truly operational. However, the German plans were well advanced and it is readily apparent now that if they had had

another year or even less the situation for the Allies would have been critical. The discussion of these German types is of considerable interest.

Experiments were likewise begun in Great Britain at an early date and an experimental jet fighter, the E28/39 flew in 1941. Although subsequent to the first German test plane, this is considered "the first *successful* airplane using the gas-turbine jet-propulsion engine."

From the British contributions the authors turn to consideration of U. S. Army work on the gas turbine and then to the U. S. Navy research. In each of the four chapters a narrative account is given of the developments that took place, with a section of technical data on some of the leading engines.

The role of the NACA in heading up government research is also discussed. Chapter seven considers some of the technical problems which still remain and the final chapter offers conjectures on the part the gas turbine engine is to play in aviation tomorrow. A chronology, a glossary, and a bibliography, which despite its heading "1941 to July 1944" includes items up to 1947, furnish useful auxiliary material.

The book is an interesting, concise report on the gas-turbine engine in aviation, suitable for those who wish to know more about its general development and problems without recourse to an exhaustive study.

G. E. PETTENGILL.

A TEXT-BOOK OF PRACTICAL ORGANIC CHEMISTRY, by Arthur I. Vogel. 1012 pages, illustrations, tables, 16 × 26 cm. New York, Longmans, Green & Co., Inc., 1948. Price, 42s.

This one-thousand page textbook is intended to meet the requirements of the student throughout the whole of his undergraduate training in chemistry. It also introduces research methods in organic chemistry and thus may serve as an intermediate reference for practicing chemists. Preparations and qualitative analysis are dealt with. A short theoretical introduction precedes each of the 600 preparations. The scope of the book may be indicated, in a general way, by a list of its chapter headings: Theory of General Technique; Experimental Technique; Aliphatic Compounds; Aromatic Compounds; Some Heterocyclic and Alicyclic Compounds; Miscellaneous Reactions; Organic Reagents in Inorganic and Organic Chemistry; Dyestuffs, Indicators and Related Compounds; Some Physiologically Active Compounds; Synthetic Polymers; Qualitative Organic Analysis. Considerable detail is given in the sections which are of particular interest to the beginner. This should provide him with a secure foundation of sound experimental technique.

H. N. MICHAEL.

A NEW NOTATION AND ENUMERATION SYSTEM FOR ORGANIC COMPOUNDS, by G. Malcolm Dyson. 63 pages, diagrams, 15 × 25 cm. London, Longmans, Green & Co., 1947. Price, $1.75.

The increasing difficulties in classification and indexing of organic compounds has led to the development of this new cipher system by Dr. Dyson. Although first formulated in 1944, it has not been published until quite extensive tests had been made with it. These tests included the ciphering of the Ring Index and five volumes of Beilstein, which resulted in no entry that did not give a distinctive cipher.

The presentation of the system is prefaced by a discussion of some of the difficulties of existing nomenclature. This new cipher employs capital letters, numerals and certain other symbols and is linear in its application. The principles of the cipher are outlined fully with examples of the various types of structure to be found and the method of ciphering them.

Enumeration is dependent upon the order of citation in the cipher for which specific and definite rules are given. Indexing of the ciphers is done by following the same order of operations used in determining enumeration, with one exception. Numerous examples of the application of the cipher are given including three hundred consecutive entries from the Ring Index, which have been ciphered and arranged in order.

One of the main features of the cipher is the fact that it lends itself to mechanical manipu-

lation. This includes computation of the molecular formula, indexing of the ciphers and searching for compounds of a predetermined structural characteristic. This adaptability to punch-card use seems one of the strongest points in its favor. What its ultimate fate will be is problematical, but it merits the consideration of chemists everywhere as an attempt to introduce simplification into organic nomenclature.

G. E. PETTENGILL.

ELECTRIC POWER TRANSMISSION, by M. P. Weinbach. 362 pages, illustrations, 16 × 24 cm. New York, Macmillan Co., 1948. Price, $5.50.

In his book, Dr. Weinbach deals with the electrical problem of transmission only. The description of generating equipment, voltage control, the mechanical suspension of the line, switching apparatus, lightning protection, etc., have been purposely omitted.

The ten chapters of this work comprehensively analyze the problem at hand. A list of chapter headings will indicate the extent of the material treated: Circuit Properties of Transmission Lines; Transmission Line Formulas; Applications of Transmission Lines; Line with Transformers; Voltage Control of Transmission Systems; Steady State Power Limits; Faulted Transmission Systems; Transient Stability; System Instability.

The rational treatment of the physical phenomena involved and of the mathematics needed for their solution makes this book highly useful to the student and practicing engineer.

H. N. MICHAEL.

PRINCIPLES OF JET PROPULSION AND GAS TURBINES, by M. J. Zucrow. 563 pages, illustrations, diagrams, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1948. Price, $6.50.

Dr. Zucrow's book is based on lecture notes used during the war in his course on jet propulsion, given at the University of California. Although observance of military security confines the book to basic principles, the attainment of the main objective, which is to present the fundamental theory of jet-propulsion engines and gas-turbine power plants, is not impaired. The book is written primarily for the student.

The first six chapters, dealing with the fundamental principles, nomenclature and energy relationships for fluids, gas thermodynamics, and airplane performance calculations, provide a firm basis for the later discussion of the continuous combustion gas turbine, the basic types of air compressors, the axial-flow turbine, the combustion chamber, the turbojet engine, and the rocket. The text is completed with a chapter on high-temperature metallurgy.

H. N. MICHAEL.

LOW-PRESSURE LAMINATING OF PLASTICS, by J. S. Hicks assisted by R. J. Francis. 162 pages, illustrations, 15 × 23 cm. New York, Reinhold Publishing Corp., 1947. Price, $4.50.

Low-pressure lamination of plastics is a field which holds promise, particularly in satisfactory accomplishments in an economical way. Manufacturing advantages involve less capital outlay, as lighter weight equipment is required. The terminology in the field of plastics is in a state of flux, but in this book, low-pressure laminating of plastics is meant to include the field of molded laminates or contoured parts as well as flat sheets, whether one ply of filler is used or several laminae of different reinforcing materials are used. Only a few of the thermosetting resins fall into this class where the curing cycle is at low pressures up to 100 psi. The book gives special attention to pressures of 0 to 15 psi.

The treatment begins with the design and use of molds, discussing materials which can be used, methods used in making them, and how separating compounds or mold lubricants are employed. The subject of what resin to use is next taken up, covering recent additions to the field of useful resins for this work. Means of hastening the curing rate with catalysts, and curing under low pressures such as evacuation of air from the envelope surrounding the mold are subsequent topics. Considerable space is devoted to reinforcements for plastics and sandwich structures, the latter meaning two sheets of laminated reinforced plastic cured with a filler or core material between them. A low density core combined with a high strength face layer gives the greatest strength to lightest weight ratio of various materials. A brief chapter on properties of plastics gives short but pointed definitions of the various properties. The latter

part of the book is devoted to descriptive illustrations of laminating—tank cover, boat deck, and milk bottle case—to show many of the versatile handling properties of low pressure laminating; joining and machining of plastics; and a discussion of product analysis involving engineering and cost principles. Appendices give an outline for laboratory experiments, a patent index, and a subject index.

Those interested in this field will find this little book of practical value.

R. H. OPPERMANN.

ELECTRONIC CIRCUITS AND TUBES, by the Electronics Training Staff at the Cruft Laboratory, Harvard University. 948 pages, illustrations, 15 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1947. Price, $7.50.

The text presented in this book was developed from the lecture notes prepared for the wartime intensified courses in electronics given to Army and Navy personnel. The material is fundamental and, therefore, should not be viewed as applicable to wartime training only. Engineering school students with the background of mathematics through calculus, of electricity and magnetism, will find the subject and its subdivisions comprehensively covered.

H. N. MICHAEL.

PHYSICS FOR STUDENTS OF SCIENCE AND ENGINEERING, by William H. Michener. 646 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1947. Price, $4.25.

This textbook is specifically written for those undergraduate students who expect to apply their knowledge of physics to science and engineering. By dealing comprehensively with the fundamental divisions of physics—mechanics, heat, wave motion and sound, light, electricity, and magnetism—Dr. Michener ably guides the student toward this goal.

The presentation is clear and pedagogically sound. The stress is on application. Many original problems are included and are carefully coordinated with the text.

H. N. MICHAEL.

SOIL MECHANICS IN ENGINEERING PRACTICE, by Karl Terzaghi and Ralph B. Beck. 566 pages, illustrations, 16 × 23 cm. New York, John Wiley & Sons, Inc., 1948. Price, $5.50.

This engineering textbook is divided into three parts dealing respectively with physical properties of soils, theoretical soil mechanics, and problems of design and construction.

The first two parts are relatively short ones, occupying together less than half of the volume, but they contain all that engineering students and the average engineer need to know about soil mechanics proper at the present time.

A short resumé of the material included in the first part would cover the physical and mechanical properties of homogeneous specimens of undisturbed and remolded soils. It deals with those properties which serve as criteria for distinguishing different soils, as well as describing them adequately. The behavior of soil masses during and after construction is also discussed.

The second part of the book introduces the reader to the theories required for solving problems involving the stability or bearing capacity of soils or the interaction between soil and water.

The third part is the "heart of the book." It is the applied part. Its initial chapter contains a discussion of the properties of natural soil deposits and the methods for investigating them. The following two are concerned with the empirical rules that pertain to the different branches of foundation and earthwork engineering. The final chapter deals with the effect on adjoining structures of such construction operations as excavating and pumping.

Because of their highly specialized nature, the designs for road and airport pavement, and for tunnel engineering are treated only referentially in the appendix.

H. N. MICHAEL.

ELEMENTARY MANUAL OF RADIO PROPAGATION, by Donald H. Menzel. 222 pages, drawings, maps, charts, nomograms, 23 × 28 cm. New York, Prentice-Hall, Inc., 1948. Price, $5.75.

When considering the importance and the expanse of the field, there are altogether too few books on radio wave propagation. Probably this is in a large part due to the ambiguous nature of the subject. Theories of ionospheric phenomena are oftimes unsubstantiated by observational evidence. On the other hand, many instances of point-to-point radio transmission are unpredictable with present-day theories. Exactly what is known and predictable about radio propagation has been collected in this one volume. Dr. Menzel makes no attempt to analyze the accuracy of the information—other than to point out the normally observed limits—nor does the author digress into the involved processes necessary to obtain this working information. The *Elementary Manual of Radio Propagation* is specifically designed for people who use the radio frequencies in communication or broadcasting. It bridges the gap between the research physicist and the radio engineer with the results of practical ionospheric theories that have withstood the test of time.

The *Manual* may be divided into three separate and distinct sections. The first is devoted to the methods of predicting the maximum usable and the lowest usable high frequencies. It will be noted that much of this material has been published previously by the IRPL (National Bureau of Standards) either in their *Radio Propagation Handbook* or in their new *Ionospheric Radio Propagation*. The second section may be said to begin with Chapter Eleven and is primarily a study of the factors involved in ground wave or direct ray transmission. Graphs show the variability in the strength of the received field relative to effective radiated power, antenna height gain and earth conductivity. The third section embraces the radio spectrum above the VHF. The material in this section is exceptionally comprehensive having been drawn from numerous sources, including the M.I.T. Radiation Laboratory Series, Vol. 13 and CRPL-T3.

To anyone familiar with the field of radio wave propagation the fact that one author in one book attempts to be all-inclusive may be rather amazing. Fortunately, however, the results vary from fair to very good. The most faults lie in the first section on sky-wave transmission. In this part of the *Manual* liberal use is made of the IRPL (later CRPL) charts and nomograms. Dr. Menzel performs a great service by reprinting and simplifying the absorption and noise grade maps which are not commonly available. At the same time this excellent intention is partially defeated by the publisher who has made the maps in the *Manual* much smaller than those currently issued monthly with the CRPL D-series Advance MUF Predictions. This necessitates that anyone desiring to use these maps must first redraw a new system of overlays, thereby subjecting his LUHF calculations to twice the normal amount of error. This same section also bears the appearance of having been prepared in great haste. Numerous small printing and proof-reading errors—such as the substitution of KM for KW—are quite common. Care should also have been exercised in coordinating the text with the nomograms in matters of symbols and terminology. The symbol $(Kd)_{50}$ is used flagrantly in the text, although the standard symbol $\overline{Kd}$ appears on most of the charts and nomograms. No mention is made of the fact that these two are identical.

In contrast, the section of the *Manual* devoted to propagation above the VHF presents a heterogeneous mass of data in a very smooth and orderly fashion. Worksheets, charts and nomograms for calculating the coverage diagrams of VHF to SHF transmitters are given. This is followed by the methods in calculating the field in the shadow zone and then under conditions of sub or superrefraction. The *Manual* ends in a three part Appendix which provides a mathematical introduction to the physical factors that cause the bending of the radio wave.

OLIVER P. FERRELL,
Radio Magazines, Inc.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

ON ENZYMES IN TUMORS.*

By Ernst Waldschmidt-Leitz, Ellice McDonald and Co-workers.

EXPERIMENTAL PART.

I. Pathologic-anatomical Analysis

(work done and reported by Wilhelm C. Hueper)

Of the two transplantable rat tumors used in the foregoing experiments, the one (Philadelphia 1) was a fibrous sarcoma, the other (Walker 256) a cellular carcinoma disposed to early and widespread necrosis. The two tumors differ biologically as well as histologically in various aspects.

Sarcoma Philadelphia 1.

The rate of growth of the tumors injected under the abdominal skin was moderate. Metastases were found only occasionally in the axillary and inguinal lymph nodes when large tumors were present. Smaller tumors were surrounded by a generally well-formed capsule of connective tissue, so that they could be taken out easily without cutting. In most of the larger tumors, the skin and muscles of the abdominal wall were infiltrated with the tumor tissue and grown onto it. The sections of the rather firm tumors were of a light pink, homogeneous, fatty appearance. In older and larger tumors there was always a more or less widespread white, fibrillar central part, which was separated rather indistinctly from the pink periphery and showed many yellow spots. The healthy tumor tissue consisted of diffuse masses or wider and narrower strands of irregularly round to oval, medium-sized tumor cells, which rarely showed division and often fibrillar projections. The surrounding stroma was a fibrous connective tissue, often of a hyalin character in the central, macroscopic whitish part of the tumor and surrounded small groups and thin strands of seemingly atrophic tumor cells. Widespread necroses were generally not present. Smaller foci of necrosis and degeneration were scattered throughout the central part in moderate numbers and were observed especially at the border between the healthy and the fibrous parts of the tumor; they were frequently surrounded and infiltrated by masses of leucocytes in small to moderately large quantities.

Carcinoma Walker 256.

The tumor showed an unusually rapid rate of proliferation; in several

* Continued from p. 453.

animals 6 to 8 weeks after inoculation the tumor was about as heavy as the rat itself. Metastases were observed to be numerous in the axillary and inguinal, but were also present in the retroperitoneal lymph nodes. Taking out these tumors without cutting was for the most part not possible, for the connective tissue capsule was relatively fragile and the tumor very soft. They were often grown fast to the abdominal wall. Ulceration of the tumors through the skin was not unusual in medium-sized and was the rule in large tumors. The healthy tumor tissue appeared pink and homogeneous on section; but even in moderately small tumors necroses of varying extent were the rule, while they were predominant in the larger tumors and constituted the greater part of the tumor, so that often only a very narrow peripheral zone of healthy tumor tissue surrounded the tumor entirely or in part. The central, necrotic part of the tumor was usually a yellowish brown, soft, cheesy, friable mass with irregular, small cavities; these contained a yellow serous fluid, which oozed freely from the section. Reddish, indistinctly outlined areas and hemorrhages were often present, especially at the junction of the necrotic and healthy tumor tissue; they also appeared as striated forms in the peripheral zone.

Histological examination gave the picture of a highly cellular carcinoma, which consisted of dense masses of large, polygonal cells with numerous mitoses. The stroma, which was generally limited, was a loose connective tissue filled with blood vessels. The brown central part was highly edematous, and contained tumor tissue in all states of degeneration and necrosis. Leucocytic infiltrations were usually present in widespread masses and were found particularly in the range of the entirely decomposed tumor; the blood vessels in the necrotic foci and in their neighborhood were often filled with thick masses of leucocytes. In tumors in which surface ulcerations were present a dense and wide leucocytic wall surrounded the cheesy necrotic area extending from the ulcer into the interior. Hemorrhagic extravasations were in abundance.

Since the foregoing work had as its aim the investigation of the quantitative and qualitative relations between certain enzymes present in tumor tissue and in their different components, the histological analysis was made, in so far as was possible, from this point of view. For this purpose, each tumor to be tested for enzymes was weighed and measurements were made of the macroscopically sound and the degenerated portions of the tumor in a section through the center; in addition, the histologic structure of the tumor was determined from a section stained with hematoxylin-phloxin. With the help of data thus obtained, the part expressed as per cent. of parenchyma, of fibrosis and of necrosis of the tumor was estimated. In the macroscopic and microscopic examinations of the tumors the presence of hemorrhage, edema, ulceration and leucocytic infiltration as well as the type of degeneration and necrosis were also noted.

II. Enzymatic Analysis.

1. Proteolytic Enzymes (work done by Arnulf Purr)

TABLE I.—*Proteolytic activity and tumor aging.*

[Tumors or parts of tumors as well as entire organs were cut up with scissors immediately after extirpation and frozen by dipping into liquid nitrogen, then pulverized by crushing with a hammer. This fine material suspended in 10 parts of 90 per cent. or extracted with 10 parts 60 per cent. glycerin; 7.0 cc. of the suspension or extract were treated with H_2S 30 min. at a pH of 7 for activation. 5.0 cc. of 8 per cent. gelatine, 1.0 cc. of *N* acetic acid, total 25.0 cc. at a pH of 4.0 for 24 hr. at 30°. The results are shown of the analysis of 10.0 cc. of the mixture (containing 2.8 cc. of enzyme solution) and signify NH_2 increase in cc. 0.05 *N* KOH. Mixture for the determination of amino-polypeptidase contained 0.25 cc. enzyme solution, 0.244 g. leucyl-di-glycine and 2.0 cc. *N*-ammonia-ammonium chloride buffer of pH 9.5, total 10.0 cc. at pH 7.8, 3 hr. at 30°. Mixture for the determination of dipeptidase contained 0.25 cc. enzyme solution, 0.188 g. leucylglycine and 2.0 cc. *N*-ammonia-ammonium chloride buffer of pH 9.5, total 10.0 cc. pH 7.8, 3 hr. 30°. Results show increase in acidity in cc. 0.05 *N* KOH. P = parenchymal, N = necrotic, F = fibrillar part in per cent.]

Tumor	No.	P	N	F	Cathepsin					
					Glycerin suspension without activation	Glycerin suspension with activation	Glycerin extract without activation	Glycerin extract with activation	Polypept. suspension*	Dipept. suspension*
Ph. 1	P 26	90	0	10	0.42	2.62	0.35	2.80	0.52	0.45
	P 22	70	0	30	1.20	2.40	0.72	2.10	0.40	0.34
	P 7	50	0	50	0.64	1.60	0.32	1.60	0.27	0.25
	P 11	40	5	55	0.65	1.44	0.37	1.44	0.38	0.32
	P 18	40	15	45	0.37	1.44	0.40	1.60	0.60	0.41
	P 4	40	20	40	0.72	1.60	—	—	0.18	0.11
	P 16	35	15	50	0.24	0.74	0.16	0.74	0.50	0.45
	P 12	35	20	45	0.74	1.30	0.48	1.44	0.68	0.55
	P 20	25	55	20	0.29	1.12	0.29	0.98	—	—
W 256	P 19	85	15	0	0.42	1.26	0.28	1.12	0.27	0.22
	P 14	80	15	5	0.37	1.16	0.37	1.40	0.23	0.18
	P 15	75	25	0	0.56	0.87	0.56	1.12	0.28	0.16
	P 17	40	20	40	0.24	0.87	0.16	1.00	0.29	0.20
	P 25	30	55	15	0.42	1.20	0.29	1.01	0.51	0.47
	P 10	20	60	10	0.32	0.74	0.37	0.96	0.55	0.40
	P 27	15	70	10	0.24	0.56	—	—	—	—
	P 24	15	75	10	0.37	0.56	0.32	0.74	0.75	0.67
	P 8	5	90	5	0.56	0.80	0.37	0.74	0.35	0.31
	P 0	0	100	0	0.12	0.24	—	—	0	0
Liver, normal					1.00	1.82	1.26	1.54	0.30	0.25
Liver, diseased	(P 4)				0.32	1.00	0.32	1.10	—	—
	(P 10)				0.65	1.50	0.65	1.40	0.55	0.40
	(P 8)				0.65	1.30	0.48	1.30	0.35	0.31
Musculature, normal					0.12	0.12	0	0	0.08	0.07
					0.15	0.22	0	0	0.06	0.04

* No notable difference was found between the enzyme content of the glycerin suspension and the glycerin extract.

2. Arginase (work done by Leopold Weil)

TABLE II.—*Arginase activity and tumor aging.*

[Preparation of tumors and organs as in Table I, finely ground material suspended (*a*) in 90 per cent. glycerin (1:10), (*b*) 1 hr. in 60 per cent. glycerin (1:10) extract. 5.0 enzyme (tumor) or 0.50 cc. (liver) or 10.0 cc. (musculature) used; for activation 2.0 cc. neutral cysteine-HCl-solution (containing 20 mg. cysteine-HCl) or 0.5 cc. 0.1 *N* $FeSO_4$, with the enzyme 1 hr. at pH 7 at 30°. 10.0 cc. of 1 per cent. arginine carbonate and 5.0 cc. 0.1 mol. glycocol buffer of pH 9.5, 60 min., at 30°. Results shown in cc. 0.02 *N*-H_2SO_4, as is customary.[1] P = parenchymal, F = fibrillar, N = necrotic part in per cent.]

Tumor	No.	N	P	F	Glycerin suspension				Glycerin extract	
					without addition	with Fe	cysteine	cyst. Fe	without addition	with cyst. Fe
Ph. 1	W 17	0	60	40	3.4	4.1	—	7.9	—	—
	W 19	0	50	50	4.2	4.4	—	6.6	—	—
	W 27	0	25	75	2.4	4.3	—	9.6	—	—
	W 13	0	15	85	3.4	6.9	4.7	6.8	2.6	4.9
	W 16	2	75	23	2.0	6.2	—	5.9	—	—
	W 26	20	40	35	4.4	13.0	—	13.3	—	5.5
	K 21	40	25	35	7.7	10.8	7.6	10.8	4.8	11.1
	W 18	60	15	20	6.1	10.5	8.2	14.5	1.9	5.6
	W 29	60	10	25	7.2	11.9	—	14.5	—	—
W 256	W 32	0	—	—	—	—	—	4.5	—	—
	W 11	15	20	65	6.3	10.4	7.3	12.3	2.6	8.4
	W 7	50	40	10	7.3	15.2	8.0	13.0	—	14.7
	W 10	65	25	10	7.2	21.5	11.3	18.2	4.1	5.6
	W 9	70	20	10	5.9	12.3	6.4	13.2	3.2	10.3
	W 28	70	30	0	7.4	12.9	—	19.3	—	—
	P 24	75	15	10	5.3	9.7	—	14.0	—	—
	P 0	100	0	0	—	—	—	17.6	—	—
Liver, normal					26.4	26.9	17.6	25.4	—	—
Liver, diseased (Ph. 1)					15.0	20.7	17.1	22.3	8.8	10.6
					11.4	21.9	—	21.0	9.4	17.0
Musculature, normal					0	—	—	0	—	—
					0	—	—	0	—	—

[1] See E. WALDSCHMIDT-LEITZ, A. SCHARIKOVA AND A. SCHÄFFNER, Hoppe-Seyler's *Zeitschrift für physiologische Chemie*, **214**: 82 (1932–33).

3. Purine Desamidases (work done by Anton Schäffner)

TABLE III.—*Desamidase activity and tumor aging.*

[Tumor Philadelphia 1. Preparation of tumors and organs as in Table I. Suspension of finely ground materials in 10 parts of 90 per cent. or extraction with 10 parts 60 per cent. glycerin. Muscle-adenylic acid or guanine used as substrate. Preparation of substrate and procedure for determinations according to G. Schmidt.[2] 1.0 cc. enzyme solution or enzyme suspension, 5 mg. adenylic acid or 4.4 mg. guanine, 2.0 cc. 0.1 mol. phosphate buffer at pH 5.9 (adenylic acid desamidase) or 10.0 cc. 0.2 mol. borate buffer at pH 8.7 (guanase), total volume 20 cc., 60 min., 30°. The results are expressed as k of the monomolecular reaction for adenylic acid desamidase and as $k = \frac{\text{cc. } 0.0033 \text{ acid}}{t \text{ (minutes)}}$ for the guanine desamidase enzyme. P = parenchymal, F = fibrillar, and N = necrotic part in per cent.]

Tumor No.	P	N	F	Adenylic acid desamidase		Guanase	
				Glycerin suspension	Glycerin extract	Glycerin suspension	Glycerin extract
S 6	75	5	20	0.0024	—	0.048	—
S 9	75	5	20	0.0022	0.0011	0.046	0.042
S 5	65	0	35	0.0024	0.0012	0.043	0.039
S 10	50	0	50	0.0025	0.0011	0.041	0.038
S 7	35	5	60	0.0011	0.0008	0.047	0.040
P 16	35	15	50	0.0013	0.0008	0.026	0.015
S 3	30	30	40	0.0009	0.0005	0.023	0.021
K 17	20	40	40	0.0008	0.0004	0.026	0.026
W 13	15	0	85	0.0013	0.0004	0.028	0.020
Liver, normal				0.0012	0.0012	0.041	0.036
				0.0012	0.0011	0.045	0.038
Liver, diseased				0.0012	0.0011	0.034	0.026
Musculature, normal				0.0120	0.0077	0.005	0.002
				0.0130	0.0080	0.005	0.002

4. Phosphatase (work done by Franz Köhler)

TABLE IV.—*Phosphatase content and tumor aging.*

[Preparation of tumors and organs as in Table I. Suspension of finely ground materials in 10 parts 90 per cent. or extraction with 10 parts 60 per cent. glycerin or removal of fat and desiccation with acetone-ether. 4.0 cc. enzyme suspension or enzyme extract or 60 mg. dry substance used. 10.0 cc. 0.1 mol. Na-glycerophosphate (corresponding to 71.6 mg. P_2O_5) containing 4.0 cc. 0.1 mol. veronal buffer [3] at pH 8.8,[4] total volume 20 cc. 3 hr., 30°. Enzyme activity stopped by addition of 5.0 cc. 5 per cent. trichloracetic acid in 10.0 cc. of the mixture. Measurement of the phosphoric acid split off after filtration was made in 10.0 cc. of the filtrates with strychnine molybdate according to G. Embden.[5] The results are expressed as k (monomol.) and are based on 10.0 cc. of trichloracetic acid—filtrate. P = parenchymal, N = necrotic, and F = fibrillar part in per cent.]

Continued on next page.

[2] G. SCHMIDT, Hoppe-Seyler's *Zeitschrift für physiologische Chemie*, **179**: 256 ff. (1928) (adenylic acid desamidase); **208**: 187 ff. (1932) (guanase).

[3] See L. MICHAELIS, *J. Biol. Chem.*, **87**: 33 (1930).

[4] The addition of magnesium salt proved to be unnecessary.

[5] G. EMBDEN, Hoppe-Seyler's *Zeitschrift für physiologische Chemie*, **113**: 138 (1921).

TABLE IV.—*Continued.*

Tumor	No.	P	N	F	Glycerin suspension	Glycerin extract	Dry substance
Ph. 1	K 24	85	0	15	0.00048	—	0.00035
	K 7	80	5	15	0.00034	0.000250	0.00015
	K 18	50	0	50	0.00021	0.000024	0.00014
	K 8	40	20	40	0.00026	0.000026	0.00013
	K 16	35	0	65	0.00026	0.000060	0.00044
	K 33	35	5	60	0.00024	—	0.00013
	W 15	35	20	45	0.00022	0.000048	0.00031
	K 36	25	0	75	0.00013	—	0.00027
	K 21	25	40	35	0.000069	0.000002	0.00001
W 256	K 22	70	5	25	0.00056	—	—
	K 19	70	15	0	0.00037	—	0.00071
	K 14	50	35	15	0.00019	0.000051	0.00016
	K 25	40	30	0	0.00022	—	0.00018
	K 12	40	45	15	0.000093	0.000054	0.000096
	K 10	40	50	10	0.000096	0.000058	0.000050
	K 11	25	60	15	0.00020	0.000022	0.00012
	K 13	25	60	15	0.00018	0.000044	0.00012
	K 35	20	35	15	0.00011	0.000036	—
	K 34	20	65	10	0.00010	—	0.00031
	K 32	15	80	5	0.000069	0.000049	0.00042
	K 31	5	70	5	0.000069	—	0.00027
Kidney, normal					0.00078	0.00032	0.00070
Kidney, diseased					0.00158	0.000070	0.00051
Musculature, normal					0.000040	0.000028	0.000033
					0.000074	0	0.000028

5. Catalase (work done by Franz Köhler)

TABLE V.—*Catalase content and tumor aging.*

[Tumor Philadelphia 1. Preparation of tumors and organs as in Table I. Suspension of finely ground materials in 10 parts 90 per cent. or extraction with 10 parts 60 per cent. glycerin. 1.0 cc. suspension or extract used. Determinations carried out as previously described.[6] Results are expressed as *k* (monomol.) P = parenchymal, F = fibrillar, and N = necrotic part in per cent.]

Tumor No.	N	P	F	Glycerin suspension	Glycerin extract
K 37	0	30	70	0.0037	0.0019
K 16	0	35	65	0.0095	0.0062
K 18	0	50	50	0.0090	0.0044
K 27	5	50	45	0.0024	0
W 26	20	35	40	0.0014	0.0031
K 30	35	15	25	0.0120	0.0049
Liver, normal				4.8*	2.8*
Liver, diseased				5.2*	2.0*
Musculature, normal				0.0042	0.0017
				0.0057	0.0035

* Determinations made with 0.025 cc., constant calculated on the basis of 1.00 cc.

[6] E. WALDSCHMIDT-LEITZ, A. SCHARIKOVA AND A. SCHÄFFNER, Hoppe-Seyler's *Zeitschrift für physiologische Chemie*, **214**: 85 (1932–33).

CURRENT TOPICS.

Dark Hours Improve Fox Fur.—Uncle Remus might have told of how Br'er Fox was like a chrysanthemum.

Such a tale would be truthful according to one of the latest discoveries in scientific fox farming, developed by U. S. Department of Agriculture specialists at the Fur Animal Experiment Station at Saratoga, N. Y. The date of prime quality in fox furs can be advanced a month or more by depriving fox pups of late afternoon sunlight in late summer and early fall.

Years ago, plant scientists of the Department discovered that chrysanthemums could be forced to bloom in advance of their natural schedule by "shortening the day"—that is, by cutting off the sunlight from the plant by shading it with black cloth, or covering it with a box, or moving it into a dark room—on a schedule which corresponded to the shortened daylight hours of autumn, its normal period of bloom. Or bloom could be retarded by giving the plants added hours of light by electricity. Commercial florists made use of this discovery in forcing earlier blooming of late-flowering varieties that were naturally too late for the big football game markets for 'mums.

In somewhat similar experiments, the fox breeding experimenters have found that the "primeness" of fox skins—the stage when they are at best market quality—is influenced by light exposure. Fox fur is normally "prime" in December. But the foxes can be deprived of some of the natural daylight by being herded into a darkened shed before sunset in the late summer and early fall—in imitation of what the hours will actually be about two months later. Under such management the fur becomes prime at an earlier date—by late October or early November—with Br'er Fox blooming like a chrysanthemum.

Commercial application of the findings is being studied further at the Station.

R. H. O.

Stopping One Crop Pest May Stop Two.—Growing knowledge of nematodes that attack plant roots is helping to clear up some crop ailments and is already modifying farm practices. More developments are bound to come as science reveals the true causes and combinations of causes of low yields.

In some cases nematodes, such as the meadow nematode or the one causing rootknot, are now known to open the way to damage by such other factors as fungi and bacteria. Or nematodes may weaken the roots so as to make the plants extremely susceptible to drought. With some plants—both weeds and crop plants—resistance to nematodes has been found important. Already the killing of nematodes with soil fumigants is becoming a farm practice on some crops and the scientists are widening the field.

Recently studies on the control of cotton wilt have shown that controlling nematodes results also in control of the wilt disease. Experimenters found that even the varieties ordinarily resistant to wilt were badly damaged by the fungus when the roots had previously been injured by the nematodes. The

plants were made vulnerable just as a man with scratches on his hands is vulnerable to many disease organisms, such as tularemia of rabbits and man.

Experimenters using a chemical that kills nematodes have increased yields of fiber greatly by ridding the cotton plants of the pests that assist the wilt, in this case the meadow nematode. After only 8 weeks the plants in fumigated rows were 3 in. taller than the untreated. At harvest there were more bolls, bigger ones, and better lint. Tests on cotton in Alabama showed that even varieties known to be wilt-resistant gave in to it when heavily attacked by nematodes. But when the nematodes were fumigated out, the wilt was cut far down, in some plots almost to the vanishing point. Fumigating with chloropicrin (tear gas) gave similar results on tomatoes suffering from wilt and rootknot nematode attack.

R. H. O.

138,000-Volt High-Pressure Oil-Filled Cable.—Installation of one of the few 138,000-volt high-pressure oil-filled cable systems for power transmission in this country has been completed in San Antonio, Texas.

The new system connects San Antonio's municipal steam-turbine power plant to the outlying Olmos and Grandview substations. These two circuits are approximately $7\frac{1}{2}$ and $3\frac{1}{4}$ miles long, respectively, the Olmos line being among the world's longest.

This type of installation consists of three insulated cables contained in an oil-filled 5-in. pipe. The pipe was shipped to San Antonio in 40-ft. lengths, which were welded to form an air-tight, underground line from the power plant to each of the substations.

The pipe laying progressed by segments of welded pipe up to 1770 ft. long. When such a segment was completed it was pressure tested for leaks, and dried. Cables were then pulled through by winches, and the section was filled with oil.

The line used for pulling the cable through the pipe was blown through by an air compressor unit. In the past, a wire has been pushed through each 40-ft. length of pipe and spliced to the preceding segment before the pipe was welded. The new method used at San Antonio enabled engineers to pull cable into continuous lengths of pipe up to one third of a mile long.

Five 10,000-gallon tank cars of oil were shipped to Texas. The special insulating oil was pumped into decontaminated and dried tank cars, and an atmosphere of nitrogen was placed on top of the oil to seal it from the air. This was the first time that insulating oil had been shipped under a controlled gas-pressure seal.

The complete San Antonio line was filled with this oil and held at a nominal pressure of 200 psi. by a permanent pumping unit installed at the power plant ends of the two lines.

R. H. O.

Traffic Congestion Increases. (*Engineering News Record*, Vol. 139, No. 18.)—There are now more cars on the road than there were at the 1941 peak and they are contributing to the biggest traffic jams in history.

The American Society of Planning Officials, commenting on plans by major cities to ease traffic congestion, reports that almost 2.6 million cars and trucks have been produced this year while considerably fewer old ones have been junked.

At Pearl Harbor time, 34,472,145 cars and trucks were licensed according to the Automotive Safety Foundation. At the end of 1946, 33,945,817 vehicles were licensed, to which has been added the bigger part of this year's auto production. Since the bulk of the demand for new cars is still unmet, traffic tangles are expected to get worse before they get better.

Removal of auto travel curbs has increased traffic congestion even more. Present traffic in Milwaukee is 92 per cent greater than during the war, while New York, Chicago, and other cities report increases of 50 per cent or more. Indicating how great the nationwide traffic increases have been, motor vehicles last year consumed 1,457 billion more gallons of gasoline than in 1941.

Not nearly so many cars are being scrapped as before the war. In 1940, 2,350,000 autos ended their service in junkyards. By 1943 that figure had dropped to 920,000. It is estimated that the 1947 total will be only about 1 million.

Traffic Congestion and Masterplans.—Washington, D. C. is among latest additions to the growing list of cities making traffic improvements a central part of long-range development plans. Plans for reducing traffic congestion are basic to masterplans newly drafted or nearing completion in Newark, Pittsburgh, Detroit, Cincinnati, Providence, San Francisco, New York, Chicago, and other major cities.

Parking is the big problem. Washington's proposed parking program calls for creation of new offstreet parking facilities to care for both short-time parking and all-day parking in congested areas. Typical of plans in other cities, the program calls for fringe parking lots in the outskirts of business districts to reduce traffic congestion on busy, over-loaded streets.

Some cities are limiting special parking privileges to ease congestion. In downtown Richmond, Va., spaces reserved at the curb for various organizations and individuals are being opened to vehicles driven by the general public. Officials expect an 800 per cent increase in the number of vehicles these spaces accommodate once they are returned to public one-hour use.

R. H. O.

Milk-Coated Cans.—In place of using milk cans coated with tin we may eventually be using containers coated with an enamel made from milk itself. Facts from the annual report of the Bureau of Dairy Industry, U. S. Department of Agriculture, suggest this.

Dairymen have commonly used as milk containers cans with a protective coating of tin that covers the sheet steel of the cans, prevents rusting, and makes it easy to clean and sterilize them.

With tin in scant supply during the war, scientists sought a substitute coating which would conserve the dwindling supply of tin. It turned out that the Federal dairy scientists developed several enamel coatings that are proving satisfactory substitutes for tin. The basic material from which these enamels can be made is lactic acid, derived from milk. Thus, milk supplies the protective coatings needed in the milk containers.

"Lactic acid," says the report, "can be converted into a viscous, rather inert, resinous material by removing the water. This resinous lactic acid, combined with oils, metals, or various substances, gives products that, when coated on metals and baked are highly resistant to water, steam, and acids, and they

are sufficiently resistant to alkalies that the coated surfaces may be washed with hot solutions of soap or other common detergents." The enamels containing an oil and the lactic acid have proved the most durable. Four patents have been issued and others are pending on these processes.

Enamels somewhat similar to these new lactic enamel coatings are familiar to housewives. They are used to coat metal cans used in processing fruits and vegetables.

R. H. O.

100,000-kw. Turbine Generator, Largest of its Type in the World.—A 100,000-kw., 3600-rpm. turbine generator unit, the first machine of its size designed for 1,000° F. steam operation, has been put into full scale service in Newark, N. J., at the Essex Generating Station of the Public Service Electric and Gas Company, New Jersey.

The machine is the largest of its type in the world, measuring 77 ft. in length and 17 ft. in width at its broadest point.

The turbine, with nineteen stages in the high-pressure casing and five stages double-flow in the low-pressure section, was designed for steam conditions of 1,250 psi., 1,000° F., 1.5-in. mercury absolute back pressure.

It exhausts through 23-in. long last-stage buckets of special warped design. The last-stage buckets, which operate at 1390 ft. per second, are mounted on a larger wheel than previously had been used on 3600-rpm. machines. The 1390 rate is approximately 10 per cent higher than previous 3600-rpm. machines were capable of attaining.

Special provisions are made for the elimination of moisture and the reduction of moisture cutting of buckets.

The turbine has a newly-developed hydraulic and lubrication oiling system with the centrifugal pump mounted on the turbine shaft fed by an oil driven booster pump in a separate oil tank in the basement.

The main generator is rated at 95,000 kw., with a power factor of 0.85 and a short circuit ratio of 0.85 at 15 psi. hydrogen pressure. It is ventilated by a new-type axial-flow fan. The rotor is cooled by a system of longitudinal cooling slots between the coil slots and by longitudinal cooling tunnels located at the bottom of the coil slots under the coils.

The set also is equipped with a 7,500-kw. direct-connected house alternator. Both generators have spring-mounted stator cores.

R. H. O.

Corn Needs Nitrogen in July.—The world shortage of nitrogen fertilizers as reported by the International Emergency Food Council lends interest to the discussion of the use of nitrogen fertilizers in an article in the current Yearbook of the U. S. Department of Agriculture. The author is Dr. F. W. Parker, assistant chief of the Bureau of Plant Industry, Soils, and Agricultural Engineering, in charge of soil and fertilizer research. The Council reports that the prospect for 1947–48 is that supplies will be only a little more than two-thirds of the requirements reported by more than 100 claimant countries—1,900,000 metric tons in prospect against requests for nearly 2,750,000 tons.

Dr. Parker reviews the rapid rise in use of nitrogen consumption in the United States—from 62,000 tons in 1900 to more than 10 times that quantity

in 1944. He also indicates the probability of a continuing rise in the demand for nitrogen—even after the world has made up the current acute shortage which has followed the undersupply during war years.

Factors such as the natural fertility of the soil, rainfall, the cropping or farming system, and the value of the crop per acre influence farm use of nitrogen, Dr. Parker points out. In the West where light rainfall limits crop production, the use of nitrogen fertilizer is not likely to prove profitable. "In a good livestock system," says Parker, "legumes are grown and much nitrogen is returned to the land in farm manure." But where high value cash crops are grown farmers are likely to use much nitrogen fertilizer.

Discussing some findings of recent research, Dr. Parker says: "Large quantities of nitrogen are required for high yields of most crops. A 60-bushel corn crop contains about 95 pounds of nitrogen, 57 pounds in the grain and 38 pounds in the stover. This nitrogen must come from the soil, legumes that have been turned under, manure, or fertilizers.

"The time when the crop needs nitrogen corresponds with its rate of growth. Little nitrogen is needed in the seedling stage, but that little is highly essential. The demand is greater when growth is quite rapid. Usually this is in midsummer for spring planted crops. Corn planted on May 22 in Ohio needed only 12 pounds of nitrogen before July 1. Between July 10 and August 10 the crop absorbed 81 pounds of nitrogen—almost 60 per cent. of the nitrogen required for the 117-bushel crop. These figures indicate the corn crop needs most of its nitrogen during the one month of maximum growth."

R. H. O.

Farm Machines Save Time.—Quicker farm transportation—of both passengers and products—is the largest single item in the record of time savings for farmers revealed in a study of the progress of farm mechanization. The tractor is ordinarily thought of as the typical symbol of mechanization, but the transportation units—the farm automobile and the motor truck—save the farmers even more time.

Comparing conditions in 1944 with those in the base period of 1917–1921, an estimated total "saving" of 4.2 billion man-hours resulted from mechanization. This takes into account the labor time that would have been required if the large 1944 production had been on the same basis as the smaller annual production 25 years earlier. A third of the saving in man-hours (1.4 billion) can be traced to gains in time resulting from use of automobiles and motor trucks. There were 3,400,000 more trucks and motor cars on farms. On the average each saved the farmer more than 400 man-hours a year, "compared with the time it would take to do the same hauling with horses and mules." All the use of the trucks was considered in making this estimate but only half the automobile use was included as farm business and the other half regarded as personal use.

Comparing these years, the time saved by tractors and tractor-drawn machines adds up to nearly a billion hours a year.

A further great time saving is in chore work—the care of farm animals. More than 13 million horses and mules disappeared from the farms in these 25 years. This meant a saving of more than a billion man-hours in chore time. This however was reduced by the hours needed for farm maintenance work on

the automobiles, motor trucks, and tractors. Of the total net savings in reduction of chore time, 760 million hours is credited to the animals displaced by the tractors and 120 million hours to the animals displaced by transportation units.

R. H. O.

Nutrition-Wise Cooking.—When a housewife's cooking is needlessly wasteful of vitamins and minerals that fresh foods offer, it may take heavy toll of what the food can contribute to health and well being. To help answer the big question: "What constitutes good cooking from a nutritive standpoint?" a group of food and nutrition specialists have systematically cooked some familiar foods in family-sized quantities.

These specialists have boiled, baked, fried, and used other common cookery methods suitable to the foods under test. Before and after cooking, they checked up on twelve nutrients, including not only vitamins sensitive to heat, air, and water, but also minerals that escape into cooking water and the ash and mineral content of the foods.

Green peas, carrots, and potatoes—three vegetables commonly prepared in a wide variety of ways—were chosen for intensive tests. The findings show, for example, that although the skin of a potato is a highly protective device for holding in nutritive values during cooking, the skin of a carrot plays no such protective role. Carrots boiled whole, pared or unpared, kept about 90 per cent of their vitamin C. Potatoes boiled whole in their skins kept practically all of their vitamin C, whereas when pared before boiling, potatoes retained from 70 to 80 per cent of this most delicate of the vitamins.

It is emphasized that these tests are part of a larger effort to learn what happens to nutrients in foods all along the line from producer to consumer.

An important and difficult part of the work has been to standardize procedures and cooking conditions, so that the same type of tests can be repeated with batches of the same food or with other foods in order to draw valid conclusions. To do this, preliminary tests were made with 20 different kinds of vegetables, cereals, breads, and meats.

R. H. O.

INDEX TO VOLUME 245.

January to June, 1948.

References with (N) are Notes from the National Bureau of Standards and references with (C) are Current Topics.

Adiabatic flow of hydrogen gas through a rocket nozzle with and without composition change (Penner and Altman), 421

Altman, D. and S. S. Penner: Adiabatic flow of hydrogen gas through a rocket nozzle with and without composition change, 421

Anisotropic metals. Strain energy in greatly deformed elastic or inelastic anisotropic engineering metals (Swainger), 501

Atomic radiation. Diamonds used to detect atomic radiation, 337(N)

Azon. Guided bombs in World War II (Grondahl), 87

Beneficiation. Iron ore supply, 360(C)

Benfield, A. E. and A. G. Emslie, H. B. Huntington, H. Shapiro: Ultrasonic delay lines II, 101

Bi-modal curves. Symmetrical bi-modal frequency curves (Gottschalk), 245

Binder, R. C. and W. V. Covert: Impact between chain roller and sprocket in a chain drive, 319

Bombing. Guided bombs in World War II (Grondahl), 87

Book Reviews:

Albert, Arthur Lemuel: Fundamental electronics and vacuum tubes, 351

American annual of photography, 1948, 349

Archibald, Raymond Clare: Mathematical table makers, portraits, paintings, busts, monuments, bio-bibliographical notes, 527

Berman, Eleanor D.: Thomas Jefferson among the arts, an essay in early American esthetics, 351

Biguenet, Charles: Les cathodes chaudes, théorie et pratique, 354

Bjork, Kenneth: Saga in steel and concrete, Norwegian engineers in America, 261

Bogue, Robert Herman: The chemistry of portland cement, 448

Brandt, Allen D.: Industrial health engineering, 263

Brown, Samuel P.: Air conditioning and elements of refrigeration, 81

Bullens, D. K.: Steel and its heat treatment, 529

Cissel, James H.: Stress analysis and design of elementary structures, 529

Crane, Theodore: Architectural construction, 260

Cushman, Frank and Robert W. Cushman: Improving supervision, 261

Custis, Eleanor Parke: Composition and pictures, 525

Davis, Harry M.: Energy unlimited. The electron and atom in everyday life, 524

Delmonte, John: The technology of adhesives, 183

Dwight, Herbert Bristol: Tables of integrals and other mathematical data, 81

Dyson, G. Malcolm: A new notation and enumerating system for organic compounds, 531

Egloff, Gustav: Physical constants of hydrocarbons, 185

Eldridge, John A.: College physics, 264

Electronic circuits and tubes, 533

Ennes, Harold E.: Broadcast operators handbook, 530

Fox, M. R.: Vat dyestuffs and vat dyeing, 186

Giesecke, Frederick, Alva Mitchell, and Henry Cecil Spencer: Technical drawing problems, 263

Gortner, Willis A. and others: Principles of food freezing, 527

Guthrie, L. O.: Factual communication: a handbook of American English, 525

Hansen, Howard J.: Modern timber design, 526

Hansen, Howard J.: Timber engineers' handbook, 350
Haynes, B. C.: Techniques of observing the weather, 263
Hicks, J. S., assisted by R. J. Francis: Low-pressure laminating of plastics, 532
Hoffmann, Banesh: The strange story of the quantum, an account for the general reader of the growth of the ideas underlying our present atomic knowledge, 187
Hougen, Olaf A. and Kenneth M. Watson: Chemical process principles, 446
Jackson, Lloyd H.: Yarn and cloth calculations, 260
James, Hubert M.: Theory of servomechanisms, 351
James, T. H. and George C. Higgins: Fundamentals of photographic theory, 526
Jones, Park J.: Petroleum production, 83
Kiefer, P. W.: A practical evaluation of railroad motive power, 447
King, Horace W. and others: Hydraulics, 529
Kirkbride, Chalmer G.: Chemical engineering fundamentals, 447
Kowalke, Otto L.: Fundamentals in chemical process calculations, 444
Kraehenbuehl, John O. and Max A. Faucett: Circuits and machines in electrical engineering, 354
Laurer, Henri, Robert Lesnick, and Leslie E. Matson: Servomechanism fundamentals, 83
Lea, D. E.: Actions of radiations on living cells, 446
Leach, W. James: Functional anatomy of the mammal, 184
Lectures delivered at Pennsylvania short course for sewage works operators, 262
Levens, Alexander S.: Nomography, 445
Li, K. C. and Chung Yu Wang: Tungsten, 183
Liapounoff, M. A.: Problème générale de la stabilité du mouvement, 527
Loeb, Leonard B.: Fundamentals of electricity and magnetism, 81
Marchand, Nathan: Ultrahigh frequency transmission and radiation, 263
Matthews, J. M. and H. R. Mauersberger: Textile fibers, 82
Meinhardt, Peter: Inventions, patents and monopoly, 529
Menzel, Donald H.: Elementary manual of radio propagation, 534
Meteorological factors in radio-wave propagation, 261
Michener, William H.: Physics for students of science and engineering, 533
Moe, Harris G.: Flight engineering and cruise control, 261
Neutron effects on animals, 524
Neville, Leslie E. and Nathaniel F. Silsbee: Jet propulsion progress. The development of aircraft gas turbines, 530
Odgers, Merle M.: Alexander Dallas Bache, scientist and educator, 352
Organic syntheses, an annual publication of satisfactory methods for the preparation of organic chemicals, 351
Pollard, L. Belle: Adult education for homemaking, 260
Pope, Alan: Wind-tunnel testing, 186
Quigley, Martin, Jr.: Magic shadows, the story of the origin of motion pictures, 350
Reddick, H. W.: Waterbury's vest-pocket handbook of engineering, 352
Richtmyer, F. K. and E. H. Kennard: Introduction to modern physics, 526
Rider, John F. and Seymour D. Uslan: Understanding vectors and phase, 525
Rogers, D. Paul: Laboratory guide for sewage works operators, 262
Seelye, Elwyn E.: Field practice, 445
Simon, Leslie E.: German research in World War II, 184
Sporn, Philip, E. R. Ambrose, and Theodore Baumeister: Heat pumps, 264
Stetson, Harlan True: Sunspots in action, 353
Terzaghi, Karl and Ralph B. Beck: Soil mechanics in engineering practice, 533
Textile brand names dictionary, 352
Trelease, Sam F.: The scientific paper, how to prepare it, how to write it, a handbook for students and research workers in all branches of science, 349
Tuska, C. D.: Patent notes for engineers, 185
Uhlig, Herbert H. (ed.): The corrosion handbook, 527
Very high-frequency techniques, 445
Vogel, Arthur I.: A text-book of practical organic chemistry, 531
Wakeman, Reginald L.: The chemistry of commercial plastics, 81

Wall, E. J. and Franklin I. Jordan: Photographic facts and formulas, 261
Warth, Albin: The chemistry and technology of waxes, 187
Weinbach, M. P.: Electric power transmission, 532
Williams, R. Tecwyn: Detoxication mechanisms, 528
Worthing, A. G. and David Halliday: Heat, 444
Yost, Don M.: The rare-earth elements and their compounds, 188
Zucrow, M. J.: Principles of jet propulsion and gas turbines, 532
Boron-steel. Boron-treated steels, 437(N)
Building construction. Temperature distribution in a test bungalow, 68(N)

Calorimetry. New flow calorimeter measures specific heats of gases, 254(N)
Carnot cycle. Thermodynamics, Part II: Work, heat and temperature concepts, and an examination of the temperature scale (Finck), 365
Carrier distillation method. Detecting impurities in uranium, 85(C)
Castings. Three-dimensional inspection of castings, 116(C)
Catapult. New airplane catapult, 364(C)
Chemistry. X-ray speeds chemical comparisons, 268(C)
Clausius principle. Thermodynamics, part I: The second law from the standpoint of the equation of state (Finck), 301
Coffin, L. F., Jr., and C. W. MacGregor, J. C. Fisher: Partially plastic thick-walled tubes, 135.
Computing machines. Institute of numerical analysis established, 342(N)
Corn needs nitrogen in July, 544(C)
Cosmic noise. Cosmic and solar radio noise, 339(N)
Counteracted gravity. Planetary properties of fast cars (Zahm), 331
Covert, W. V. and R. C. Binder: Impact between chain roller and sprocket in a chain drive, 319
Crop pests. Stopping one crop pest may stop two, 541(C)

Decius, J. C.: Dimensional analysis: An approach from transformation theory and a criterion for scaling model experiments, 379
Delay lines. Ultrasonic delay lines. I (Huntington, Emslie and Hughes), 1
Ultrasonic delay lines. II (Emslie, Huntington, Shapiro, and Benfield), 101
Diamonds used to detect atomic radiation, 337(N)
Diesel engines. Stand built to test governors under service conditions, 85(C)
Dimensional analysis: An approach from transformation theory and a criterion for scaling model experiments (Decius), 379
Direct current measurement. Visual measurements of short pulses of direct current (Ordung), 37
Dust. A photometric investigation of dust (Spencer and Malkiel), 389
Dyes. Heat-resistant dyes, 420(C)

Elasticity. Applications of tensor analysis to elasticity and piezoelectricity (Jurmain), 475
Partially plastic thick-walled tubes (MacGregor, Coffin, and Fisher), 135
Strain energy in greatly deformed elastic or inelastic anisotropic engineering metals (Swainger), 501
Electric furnace brazing. Electric furnace brazing facilitates manufacture of insecticide bombs, 361(C)
Electric locomotives. Electric locomotives for Paulista railway of Brazil, 454(C)
Electrodeposition of tungsten alloys, 435(N)
Electrolysis. Electronic power source for electrolysis, 433(N)
Electronics. Electronic power source for electrolysis, 433(N)
Electropult. New airplane catapult, 364(C)
Emslie, A. G., and H. B. Huntington, H. Shapiro, A. E. Benfield: Ultrasonic delay lines. II, 101
Emslie, A. G., and H. B. Huntington, V. W. Hughes: Ultrasonic delay lines. I, 1
Entropy. Thermodynamics, Part I: The second law from the standpoint of the equation of state (Finck), 301
Equations of state. Thermodynamics, part I: The second law from the standpoint of the equation of state (Finck), 301
Thermodynamics, part II: Work, heat, and temperature concepts, and an examination of the temperature scale (Finck), 365

Farm machinery. Farm machines save time, 545(C)
Fertility. The drain on fertility, 269(C)
Fertilizer. Corn needs nitrogen in July, 544 (C)
The drain on fertility, 269(C)
Rising fertilizer tonnage, 226(C)
Finck, J. L.: Thermodynamics, part I: The second law from the standpoint of the equation of state, 301
Thermodynamics, part II: Work, heat and temperature concepts, and an examination of the temperature scale, 365
Fire control. Radio proximity fuze in fire fighting, 341(N)
Fisher, J. C., and C. W. MacGregor, L. F. Coffin, Jr.: Partially plastic thick-walled tubes, 135
Flow. Adiabatic flow of hydrogen gas through a rocket nozzle with and without composition change (Penner and Altman), 421
Food preparation. Nutrition-wise cooking, 546(C)
Franklin Institute:
Biochemical Research Foundation Notes:
Elegy written in (and for) a city laboratory shortly before moving to the country, 181
Leitch, James L.: Some factors in the Nessler method of nitrogen determination, 355
Waldschmidt-Leitz, Ernst, Ellice McDonald and co-workers: On enzymes in tumors, 449, 535
Zittle, Charles A.: Enzymatic hydrolysis of desoxyribonucleic acid prepared with the use of strong sodium hydroxide, 78
Zittle, Charles A.: Manometric determination of phosphomonoesterase (alkaline phosphatase), 265
By-laws, 172
Committee on Science and the Arts: Abstracts of Proceedings, November 12, 1947, 75; December 10, 1947, 76; January 14, 1948, 178; March 10, 1948, 345; May 12, 1948, 521
Library. Publications received: 84, 188, 259, 348, 443, 524
Library Notes: 76, 178, 258, 346, 441, 522
New members elected June 1, 1947 to December 1, 1947, 74
Stated Meetings: Minutes, December 17, 1947, 73; January 21, 1948, 171; February 18, 1948, 257; March 17, 1948, 345; April 21, 1948, 441; May 19, 1948, 521
Fredholm's equations. A mechanical integraph for the numerical solution of integral equations (Tea), 403
Frequency curves. Symmetrical bi-modal frequency curves (Gottschalk), 245
Frozen food. 50 per cent of perishables may be frozen by 1960, 362(C)
Frozen juice like fresh, 52(C)
Fungi. Spores by the billion, 318(C)
Fur. Dark hours improve fox fur, 541(C)

Gas. New flow calorimeter measures specific heats of gases, 254(N)
Gas turbines. Experimental gas turbines being constructed, 134(C)
Materials for gas turbine service, 456(C)
Gasometer. Pressure gasometer aids in industrial health studies, 455(C)
Gauss curves. Symmetrical bi-modal frequency curves (Gottschalk), 245
Gears. Impact between chain roller and sprocket in a chain drive (Binder and Covert), 319
Gel strength. Mercury-balance method for the measurement of gel strength, 71(N)
Generating stations. Trend of power transmission economics toward operation of large generating stations at high circuit loadings, 134(C)
Generators. New 108,000-kw. generator for Grand Coulee, 388(C)
100,000-kw. turbine generator, largest of its type in the world, 544(C)
Gerszonowicz, S.: Rigorous and approximate treatment of long line transmission problems by hyperbolic functions, 53
Governors. Stand built to test governors under service conditions, 85(C)
Gottschalk, V. H.: Symmetrical bi-modal frequency curves, 245
Green's function. An integral-equation approach to problems of vibrating beams. Part I (White), 25; Part II, 117
Grondahl, L. O.: Guided bombs in World War II, 87
Guided missiles. Guided bombs in World War II (Grondahl), 87

Hardenability. Boron-treated steels, 437(N)
Heat. Temperature distribution in a test bungalow, 68(N)
Holmes, M. C.: Insulation variability, 159
Home economics. Nutrition-wise cooking, 546(C)
Housing. Assembly line produces house an hour, 318(C)
Hughes, V. W., and H. B. Huntington, A. G. Emslie: Ultrasonic delay lines. I, 1
Huntington, H. B., and A. G. Emslie, V. W. Hughes: Ultrasonic delay lines. I, 1
Huntington, H. B., and A. G. Emslie, H. Shapiro, A. E. Benfield: Ultrasonic delay lines. II, 101
Hydrodynamics. Dimensional analysis: An approach from transformation theory and a criterion for scaling model experiments (Decius), 379
Hydrogen gas. Adiabatic flow of hydrogen gas through a rocket nozzle with and without composition change (Penner and Altman), 421
Hyperbolic functions. Rigorous and approximate treatment of long line transmission problems by hyperbolic functions (Gerszonowicz), 53

Impact testing. Boron-treated steels, 437 (N)
Industrial health. Pressure gasometer aids in industrial health studies, 455(C)
Insecticides. Electric furnace brazing facilitates manufacture of insecticide bombs, 361(C)
Inspection. Sound waves inspect metal, 226(C)
Three-dimensional inspection of castings, 116(C)
Insulation. Insulation variability (Holmes), 159
Integral equations. An integral-equation approach to problems of vibrating beams. Part I (White), 25; Part II, 117
A mechanical integraph for the numerical solution of integral equations (Tea), 403
Integraph. A mechanical integraph for the numerical solution of integral equations (Tea), 403
Invention. A study of patent policies in educational institutions, giving specific attention to the Massachusetts Institute of Technology (McKusick), 193, 271
Ionosphere. Automatic ionosphere recorder, 164(N)
Iron ore. Iron ore supply, 360(C)
Isotopes. Detection of isotopes by tracer micrography, 67(N)
Ives, Ronald L.: Meteorological conditions accompanying mirages in the Salt Lake Desert, 457

Jet engines. Problems to face in testing jet engines, 454(C)
Jurmain, Jacob Harry: Applications of tensor analysis to elasticity and piezoelectricity, 475

Kelvin-Planck principle. Thermodynamics, part I: The second law from the standpoint of the equation of state (Finck), 301

Lactic enamel. Milk-coated cans, 543(C)
Light waves. Light wave of mercury 198 as the ultimate standard of length, 166(N)
Log barker. Pneumatic barker cuts log waste, 474(C)
Lo-Ho: Construction of alignment nomogram from empirical data, 227
Long line transmission. Rigorous and approximate treatment of long line transmission problems by hyperbolic functions (Gerszonowicz), 53
Luminaires. A photometric investigation of dust (Spencer and Malkiel), 389

MacGregor, C. W., and L. F. Coffin, Jr., J. C. Fisher: Partially plastic thick-walled tubes, 135
Malkiel, Selma and Domina Eberle Spencer: A photometric investigation of dust, 389
Man-made weather. Expanding air causes sudden temperature drop, 189(C)
McKusick, Vincent Lee: A study of patent policies in educational institutions, giving specific attention to the Massachusetts Institute of Technology, 193, 271
Measurement. Automatic ionosphere recorder, 164(N)
Light wave of mercury 198 as the ultimate standard of length, 166(N)
Measurement of total stratosphere ozone, 253(N)
New process control instruments, 363(C)

Mechanical resonance. An integral-equation approach to problems of vibrating beams. Part I (White), 25; Part II, 117
Mercury-balance method. Mercury-balance method for the measurement of gel strength, 71(N)
Metal powders. Particle-size distribution in powder metallurgy, 517(N)
Meteorology. Meteorological conditions accompanying mirages in the Salt Lake Desert (Ives), 457
Mirages. Meteorological conditions accompanying mirages in the Salt Lake Desert (Ives), 457

Nematodes. Stopping one crop pest may stop two, 541(C)
Nomograms. Construction of alignment nomogram from empirical data (Lo-Ho), 227
Numerical analysis. Institute of numerical analysis established, 342(N)

Oil. X-raying oil fields, 52(C)
Oil-filled cable. 138,000-volt high-pressure oil-filled cable, 542(C)
Ordung, Philip F.: Visual measurements of short pulses of direct current, 37
Oscillographic measurement. Visual measurements of short pulses of direct current (Ordung), 37

Particle-size distribution in powder metallurgy, 517(N)
Patents. A study of patent policies in educational institutions, giving specific attention to the Massachusetts Institute of Technology (McKusick), 193, 271
Penner, S. S. and D. Altman: Adiabatic flow of hydrogen gas through a rocket nozzle with and without composition change, 421
pH. Standardization of the pH scale, 163 (N)
Photography. Picture, made at millionth of second exposure, developed for projection in 30 seconds, 100(C)
Photometry. A photometric investigation of dust (Spencer and Malkiel), 389
X-ray speeds chemical comparisons, 268 (C)
Piezoelectricity. Applications of tensor analysis to elasticity and piezoelectricity (Jurmain), 475
Planetary travel. Planetary properties of fast cars (Zahm), 331
Plasticity. Partially plastic thick-walled tubes (MacGregor, Coffin, and Fisher), 135
Plastics. Home-plumbing studies use plastic pipe, 343(N)
Plumbing. Home-plumbing studies use plastic pipe, 343(N)
Pneumatics. Pneumatic barker cuts log waste, 474(C)
Portable houses. Assembly line produces house an hour, 318(C)
Powder metallurgy. Particle-size distribution in powder metallurgy, 517(N)
Power source. Electronic power source for electrolysis, 433(N)
Power transmission. 138,000-volt high-pressure oil-filled cable, 542(C)
Trend of power transmission economics toward operation of large generating stations at high circuit loadings, 134(C)
Protective coatings. Milk-coated can, 543 (C)
Proximity fuze. Radio proximity fuze in fire fighting, 341(N)

Quackgrass. Plant scientists report on chemical quackgrass killer, 192(C)

Radiant heat. Oregon installs radiant-heated road, 363(C)
Radiation. Diamonds used to detect atomic radiation, 337(N)
Light wave of mercury 198 as the ultimate standard of length, 166(N)
Ultrasonic delay lines. I (Huntington, Emslie and Hughes), 1
Ultrasonic delay lines. II (Emslie, Huntington, Shapiro, and Benfield), 101
Razon. Guided bombs in World War II (Grondahl), 87
Research. A study of patent policies in educational institutions, giving specific attention to the Massachusetts Institute of Technology (McKusick), 193, 271
Roads. Oregon installs radiant-heated road, 363(C)
Rocket nozzle. Adiabatic flow of hydrogen gas through a rocket nozzle with and without composition change (Penner and Altman), 421

Roller-sprocket impact. Impact between chain roller and sprocket in a chain drive (Binder and Covert), 319

Safety devices. Airport snow removal, 388 (C)
Problems to face in testing jet engines, 454(C)
Safety engineering. X-ray protection, 269 (C)
Scale model. Dimensional analysis: An approach from transformation theory and a criterion for scaling model experiments (Decius), 379
Shapiro, H., and A. G. Emslie, H. B. Huntington, A. E. Benfield: Ultrasonic delay lines. II, 101
Short pulses of direct current. Visual measurements of short pulses of direct current (Ordung), 37
Snow removal. Airport snow removal, 388 (C)
Solar noise. Cosmic and solar radio noise, 339(N)
Soya sauce. Science captures a Chinese art, 24(C)
Space car. Planetary properties of fast cars (Zahm), 331
Specific heat. New flow calorimeter measures specific heats of gases, 254(N)
Spectrophotometry. A photometric investigation of dust (Spencer and Malkiel), 389
Spencer, Domina Eberle and Selma Malkiel: A photometric investigation of dust, 389
Spores. Spores by the billion, 318(C)
Sprocket and chain. Impact between chain roller and sprocket in a chain drive (Binder and Covert), 319
Static. Cosmic and solar radio noise, 339 (N)
Strain. Strain energy in greatly deformed elastic or inelastic anisotropic engineering metals (Swainger), 501
Stratosphere. Measurement of total stratosphere ozone, 253(N)
Strength-distribution curves. Insulation variability (Holmes), 159
Stress. Strain energy in greatly deformed elastic or inelastic anisotropic engineering metals (Swainger), 501
Partially plastic thick-walled tubes (MacGregor, Coffin, and Fisher), 135
Structure of metals. Fundamental properties of metals, 330(C)
Sugar. More sugar from graded fields, 190 (C)
Supersonics. Sound waves inspect metal, 226(C)
Surface-fitting. Construction of alignment nomogram from empirical data (Lo-Ho), 227
Swainger, K. H.: Strain energy in greatly deformed elastic or inelastic anisotropic engineering metals, 501
Symmetry. Symmetrical bi-modal frequency curves (Gottschalk), 245
Synthetic fibers. Synthetic fibers other than rayon increasing rapidly in importance, 190(C)

Tea, Peter L.: A mechanical integraph for the numerical solution of integral equations, 403
Temperature scale. Thermodynamics, part II: Work, heat, and temperature concepts, and an examination of the temperature scale (Finck), 365
Tensor analysis. Applications of tensor analysis to elasticity and piezoelectricity (Jurmain), 475
Textiles. Heat-resistant dyes, 420(C)
Thermodynamics, part I: The second law from the standpoint of the equation of state (Finck), 301
Thermodynamics, part II: Work, heat, and temperature concepts, and an examination of the temperature scale (Finck), 365
Titanium. New process forecasts wider use of titanium, 330(C)
Tracer micrography. Detection of isotopes by tracer micrography, 67(N)
Traffic congestion increases, 542(C)
Transducers. Ultrasonic delay lines. I (Huntington, Emslie and Hughes, 1
Ultrasonic delay lines. II (Emslie, Huntington, Shapiro, and Benfield), 101
Transformation theory. Dimensional analysis: An approach from transformation theory and a criterion for scaling model experiments (Decius), 379
Transmission. Rigorous and approximate treatment of long line transmission problems by hyperbolic functions (Gerszonowicz), 53